Time-Consistently Undominated Policies

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Introduction

- A paper about Kydland & Prescott (1977) problems
  - Expectations of future policies affect current choice set
  - Benefits to making binding **promises**, *ex-post* incentives to break them
- $\Rightarrow$ large class of policy design problems in macro

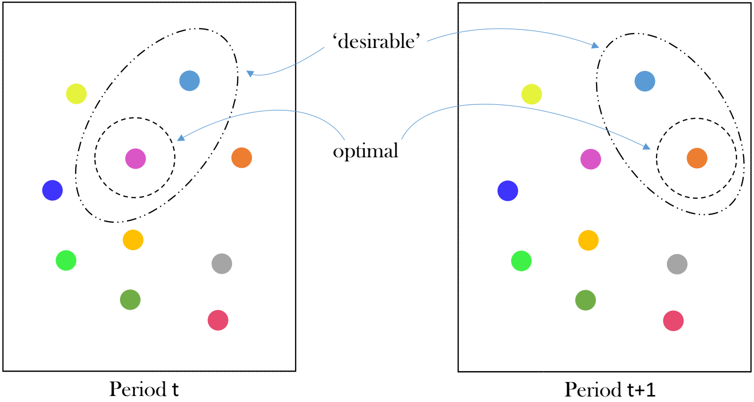
What we do

Sustained desirability

- Time inconsistency ⇔ no policy sequence lives in argmax set forever
- Normative policy design non-trivial
- **Ramsey approach**: best policy for initial period
  - ~~Recursive~~ argmax
- **This paper**: a desirable policy that remains desirable?
  - Recursive ~~argmax~~

What we do

An abstract visual



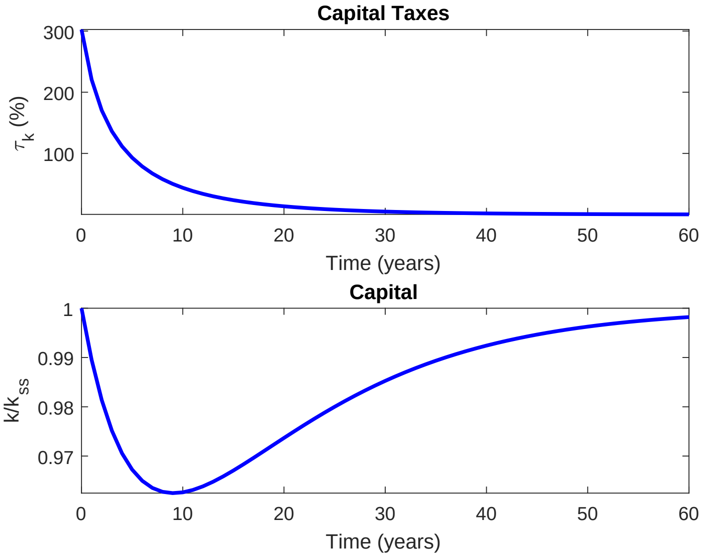
What we (don't) do

Axiomatic approach

- Construction of desirable set will be **axiomatic**
- We **assume commitment**, and do **not** consider non-cooperative implementation
- Analogy to cooperative solutions in game theory:
  - *Nash (1950), Shapley (1953)*
- *'If I want to commit to a plan that I can justify in the same way in each period, what would it look like?'*

Why do this?

1. Date-contingent Ramsey policy



Why do this?

1. Date-contingent Ramsey policy

- Designation of unique ‘period zero’ seems arbitrary
- Woodford (2003): need *“systematic decision procedure in the light of which ... current actions are always to be justified”*

Why do this?

2. Long-run outcomes

- Long-run Ramsey outcomes can be extremely undesirable...
  - ***Immiseration*** *results (Thomas & Worrall, 1990, dynamic Mirrlees, ...)*
  - *'Corner' outcomes for optimal cap. taxes (Straub & Werning, 2015)*
- But steady-state policy common focal point *because* of transition issue!
  - *Woodford (1999): 'timeless perspective'*
  - *Capital taxes again...*

Problem setup

Constraints

- Will think about general problem of choosing **allocations** $\{a_t, x_t\}_{t \geq s}$
- Feasible set of these from s on: Ξ_s
- Includes forward-looking constraints for $t \geq s$:

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} h\left(a_{t+\tau}\right) \right] \geq h^0\left(a_t\right)$$

- [Many applications map to this framework]

Problem setup

Promises

- Forward-looking constraints can be rewritten in terms of **promises**:

$$h(a_t) + \beta \omega_{t+1} \geq h^0(a_t)$$

$$\mathbb{E}_{t-1}\left[h(a_t) + \beta \omega_{t+1}\right] \geq \omega_t$$

- '**Promise making**' & '**Promise keeping**'
- Given $\{\omega_t\}_{t \geq s}$, choice is time-consistent...

Problem setup

Preferences and choice

- Policymaker in s has standard (time-additive) prefs over Ξ_s:

$$W_s = \mathbb{E}_s \left[\sum_{t=s}^{\infty} \beta^{t-s} r\left(a_t\right) \right]$$

- Want a **desirable set** $D_s \subset \Xi_s$ s.t. for some $\{a_t^*, x_t^*\}_{t \geq s}$:

$$\{a_t^*, x_t^*\}_{t \geq r} \in D_r$$

  for all $r \geq s$...
- ... with D_s endowed with appealing normative properties

Choice

Dominated selections

- Choice from Ξ_s *as a whole* is time-inconsistent...
- ... But some comparisons in Ξ_s straightforward
- Rule out 'dominated' choices from D_s

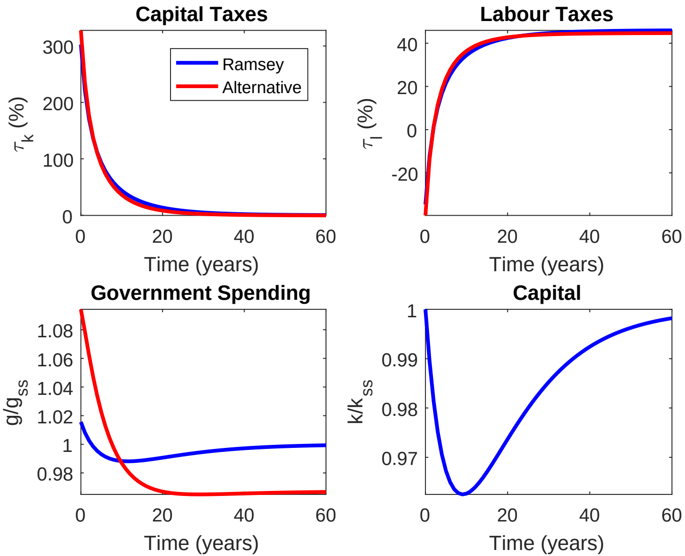
Choice

Dominated selections

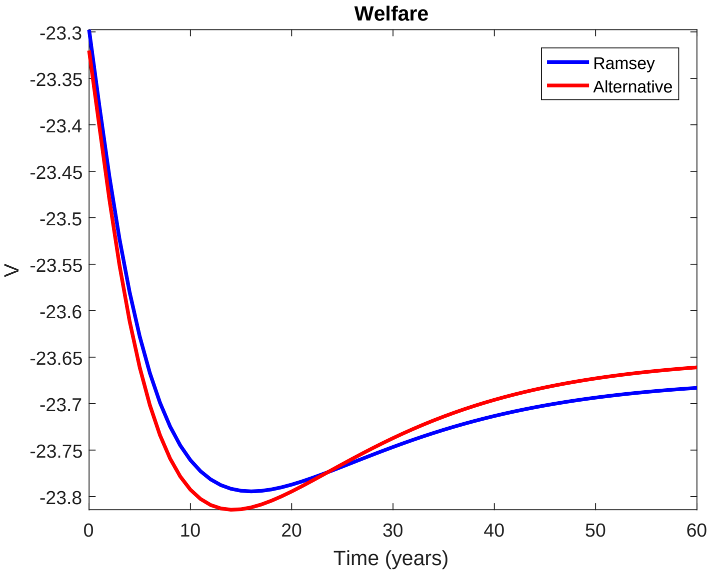
Two particular restrictions on D_s:

1. **Time-consistent sets**:
   - Suppose $S \subset \Xi_s$ s.t. choice in S alone is time-consistent
   - $\Rightarrow$ sub-optimal choices in S should not be in D_s
   - *Arises when allocations deliver same* $\{\omega_t\}_{t \geq s}$ *over time*
2. **Time-invariant rankings**:
   - Suppose $\{a'_t, x'_t\}_{t \geq r} \succ \{a''_t, x'_t\}_{t \geq r}$ *for all* $r \geq s$
   - $\Rightarrow \{a''_t, x'_t\}_{t \geq s}$ should not be in D_s
   - *Pareto principle over time, w restricted domain*

Condition 2: example



Condition 2: example



Analysis

Value function representation

- Conditions give **incomplete ordering** on Ξ_s in all s
- *So far, so abstract...*
- Condition 1 $\Rightarrow$ value function representation:

$$V\left(\{\omega_t\}_{t\geq s}, x_{s-1}\right) := \max\left(W_s\right), \text{given promises}$$

- Promises remain to be chosen

Analysis

Equivalence

- Main result for making all this tractable:

  TCUP $\Rightarrow \{\omega_t\}_{t \geq s}$ *optimal* ***every*** *period for* ***one*** *dimension*

- $\Rightarrow \{\omega_t\}_{t \geq s} = \{\bar{\omega}_t + \delta_t \theta\}$, optimal choice of θ independent of s
- With extra conditions, have '$\Leftarrow$' too
- *Ramsey:* $\{\omega_t\}_{t \geq s}$ *optimal in* ***one*** *period along* ***every*** *dimension*

Analysis

Characterisation

- ⇒ TCUP will have simple characterisation, based on FOCs
- Expressed in terms of **multipliers on FL constraints**
  - *C.f. Marcet & Marimon (1998, 2016)*
- Can replace Ramsey condition with this, and solve as usual...
- Interpretation of θ: single policy lever, agreed on in all periods
- Possible $\{\delta_t\}$ restricted, but not unique
- We propose **symmetry** in $\{\delta_t\}$ through time as a refinement – example-specific meaning

Analysis

Characterisation (no shocks)

- Let λ_t^m be multiplier on promise-making, λ_t^k on promise-keeping
- Ramsey policy sets $\lambda_{-1}^k = \lambda_{-1}^m = 0$ and:

$$\lambda_t^k = \lambda_{t-1}^k + \lambda_{t-1}^m$$

- Our policy sets:

$$\sum_{t=s}^{\infty} \beta^{t-s} \left[\delta_t \lambda_t^k - \delta_{t+1} \beta \left(\lambda_t^k + \lambda_t^m \right) \right] = 0$$

and also at $s+1$, so:

$$\delta_s \lambda_s^k = \delta_{s+1} \beta \left(\lambda_s^k + \lambda_s^m \right)$$

Analysis

Resolving symmetry

- $\{\delta_t\}$ gives marginal effect of policy lever on promises at each point in time
- 'Symmetric' choice could mean ...
  - *Identical **marginal** effect at each point in time:* $\delta_t = \bar{\delta}$
  - *Identical **proportional** effect at each point in time:* $\delta_t = \bar{\delta}\omega_t$
- Appropriate choice depends on the economics!
- Contrast utility & wealth promises...
- In general: what transformations should policy be invariant to?

Outcomes

Stable multipliers with shocks

- Common feature of our approach: changes to multipliers no longer fully persistent
- E.g. social insurance model, agent with shock history σ:

$$\lambda_t^k(\sigma) = \beta\left(\lambda_t^k(\sigma_-) + \lambda_t^m(\sigma)\right)$$

- Ramsey:

$$\lambda_t^k(\sigma) = \lambda_{t-1}^k(\sigma_-) + \lambda_{t-1}^m(\sigma)$$

- Big implications for evolution of wealth distn in response to shocks

Outcomes

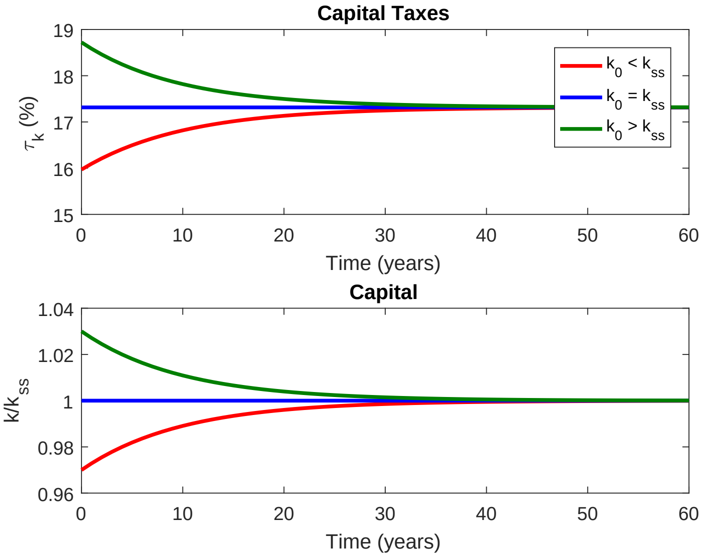
Properties

Resulting policies have many appealing properties:

- **Optimal constant selection** in models w/out states
- Standard limits as time inconsistency disappears
- Simple, intuitive restrictions on wedges in Ramsey tax models
- Stable wealth distn in social insurance models (**no immiseration**)
- Far **simpler** policy dynamics...

Outcomes

TCUP: simple dynamics (Judd problem)



A simple example

- Consider textbook NK '**inflation bias**' problem
- Policymaker's loss in s:

$$L_s := \sum_{t=s}^{\infty} \beta^{t-s} \left[\pi_t^2 + \chi \left(y_t - \bar{y} \right)^2 \right]$$

- PC constraint:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma y_t$$

- Time-inconsistency:
  - Promise of low π_{t+1} benefits policymaker in t (and earlier)
  - Policymaker in $t+1$ will want to deviate

A simple example

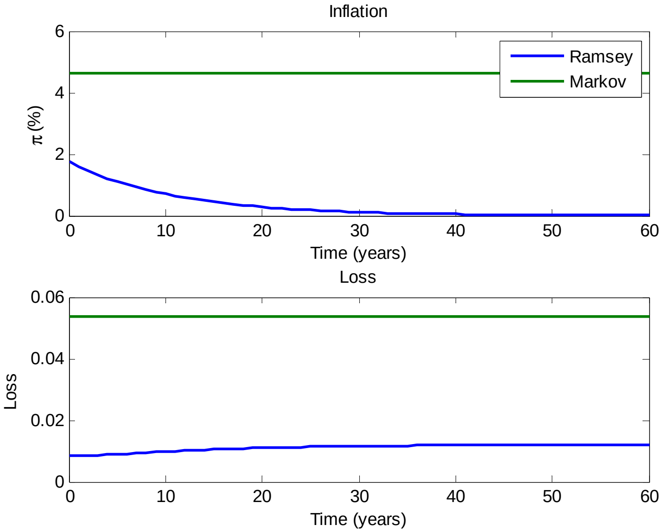
- Simplicity ⇒ **constraint dominance** not relevant here
- Promises ↔ inflation
- Given inflation sequence, only one allocation possible...

$$\bar{\pi}_t = \beta \bar{\pi}_{t+1} + \gamma y_t$$

- **Preference dominance** illustrated by comparing Ramsey & Markov policies

A simple example

Ramsey and Markov policy



A simple example

Preference dominance

- $\{\pi_t^R, y_t^R\}_{t=s}^{\infty} \succ \{\pi_t^M, y_t^M\}_{t=s}^{\infty}$ for all $s \geq 0$
- 'Ramsey policy pref-dominates Markov policy for all $s \geq 0$'
- Ramsey policy itself cannot be pref-dominated in period 0...
- ... But its continuation?

A simple example

Preference dominance

- Optimal constant policy solves:

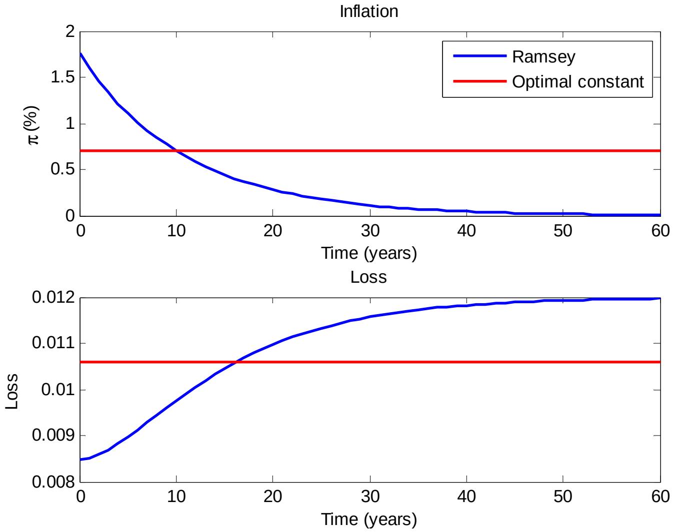
$$\arg\min_{\{\pi^c, y^c\}} \sum_{t=s}^{\infty} \beta^{t-s} \left[(\pi^c)^2 + \chi (y^c - \bar{y})^2 \right]$$

$$s.t.\ \pi^c = \beta \pi^c + \gamma y^c$$

- Clearly this will (always) pref. dominate Markov
- Comparison with Ramsey?

A simple example

Ramsey and optimal constant policy



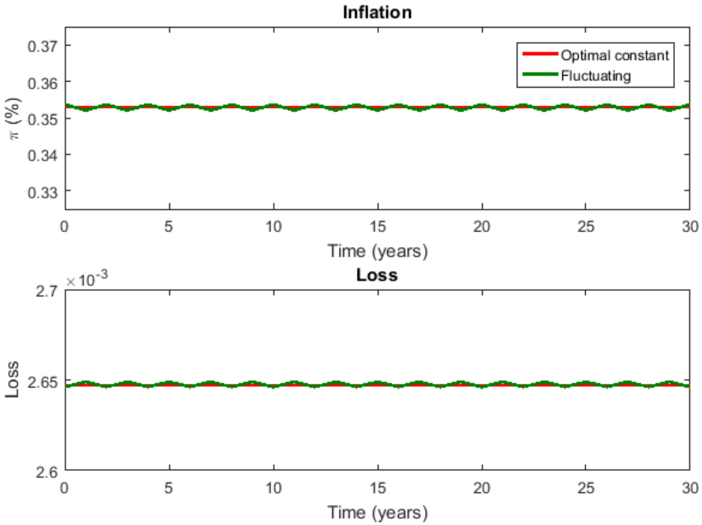
A simple example

Preference dominance

- In period 0, neither policy pref-dominates other
  - *[Both are in D]*
- By period 17, optimal constant policy pref-dominates Ramsey **continuation**
- ⇒ Ramsey starts out undominated, but is not **recursively undominated**
- Are there other recursively undominated policies? Yes, but...

A simple example

Preference dominance



A simple example

Recursive desirability and time consistency

- Optimal constant policy has a dual interpretation as a **time-consistent** choice
- Solution to:

$$\min_{\{\pi^c, y^c\}} \sum_{t=s}^{\infty} \beta^{t-s} \left[(\pi^c)^2 + \chi (y^c - \bar{y})^2 \right]$$
$$s.t.\ \pi^c = \beta \pi^c + \gamma y^c$$

  is independent of s
- Optimal along **one dimension** in **every period**
  - *[C.f. Ramsey: optimal along every dimension in one period]*
  - *[C.f. Markov: have control over future policy!]*
- Key to operationalising approach

A simple example

Recursive desirability and time consistency

- More generally, any recursively desirable policy **time-consistently** (necessarily) solves problem of form:

$$\min_{\theta \in \mathbb{R}} \sum_{t=s}^{\infty} \beta^{t-s} \left[(\bar{\pi}_t + \delta_t \theta)^2 + \chi (y_t - \bar{y})^2 \right]$$

$$s.t. \; (\bar{\pi}_t + \delta_t \theta) = \beta (\bar{\pi}_{t+1} + \delta_{t+1} \theta) + \gamma y_t$$

  - *[Given $\{\bar{\pi}_t, \delta_t\}$, $\{\delta_t\}$ bounded (&...)]*
- Optimal along the dimension traced out by θ in every s

A simple example

Time-consistent optimality conditions

- Let $\{\lambda_t\}$ be constraint multipliers in previous problem
- Optimising wrt θ in period s gives:

$$\sum_{t=s}^{\infty} \left[\delta_t \left(-\lambda_t + \pi_t\right) + \beta \delta_{t+1} \lambda_t\right] = 0$$

- But **time consistency** implies same holds for $s+1$, so:

$$\delta_s \left(-\lambda_s + \pi_s\right) + \beta \delta_{s+1} \lambda_s = 0$$

A simple example

Symmetry

- Clearly **optimal constant policy** of particular interest here
- Can be recovered by applying previous technique, and requiring δ_s constant in all periods:

$$\lambda_s = \beta\lambda_s + \pi_s$$

- C.f. **Ramsey** (for $s > 0$):

$$\lambda_s = \lambda_{s-1} + \pi_s$$

- If higher inflation today means higher inflation tomorrow...

A simple example

Symmetry: some intuition

- View choice over θ as a '**policy lever**', marginal effects $\{\delta_t\}$
- Constant $\delta_t \leftrightarrow$ '*Identical power to act on economic variables through time*'
- Time-consistent optimality in choice of a symmetric lever
- This idea of symmetry will generalise

A simple example

Limiting departures from Ramsey

- Another instructive restriction is satisfied by recursively undominated policy
- Ramsey policymaker prior to t wants:

$$\lambda_t = \lambda_{t-1} + \pi_t$$

- Ramsey policymaker in t wants:

$$\lambda_t = \pi_t$$

- Recursively undominated policy from s has:

$$\lambda_t \in (\pi_t, \lambda_{t-1} + \pi_t)$$

  a.e. for large enough t