Time-Consistent Institutional Design

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Introduction

- This is a normative paper about Kydland & Prescott (1977) problems
  - Expectations of future policies affect current choice set
  - Benefits to making binding promises, *ex-post* incentives to break them
- Everywhere in macro:
  - Capital taxation, inflation bias, dynamic social insuarance, human capital policies, ...
- Our focus is general, but without aggregate risk

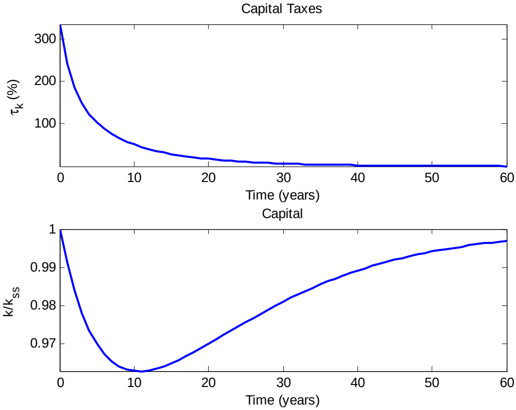
What we do

A new way to choose desirable policies (promises)

- Time inconsistency ⇔ no policy is recursively optimal
- Ramsey: impose initial prefs
  - ~~Recursive~~ optimality
  - [Recursive *implementation* possible, via expanded state space]
- This paper: a desirable policy that remains desirable
  - Recursive ~~optimality~~
  - Pareto criterion, applied across time
  - Applied exclusively to choice of promises (c.f. Farhi & Werning, 2007)
- Assume policymaker has commitment device – ‘institutional design’

Why do this?

- Main motivation: *Ramsey policy does not seem to answer practical policy questions*
- Two features commonly viewed as 'problematic':
  1. Substantial disparity btwn long & short run choices
     - *E.g.: Variant of Judd (1985):*



Why do this?

- Main motivation: *Ramsey policy does not seem to answer practical policy questions*
- Two features of Ramsey policy often viewed as 'problematic':
  1. Substantial disparity btwn long & short run choices
  2. Long-run outcomes may not be desirable *per se*
     - ***Immiseration*** *results (Thomas & Worrall, 1990, dynamic Mirrlees, ...)*
     - *Optimal cap. taxes* ⇒ *'corner' outcomes (Straub & Werning, 2015)*
- *If I can* find a 'recursively desirable' policy, can deal with both

Related work

- Important works responding to these concerns, but quite context-specific:
  - Long run desirability/immiseration:
    - Phelan (2006), Farhi & Werning (2007): change Pareto weighting on different generations
    - Sleet & Yeltekin (2006), Golosov & Iovino (2015): sustainable plans ⇒ endogenous social discounting
  - Short run/long run disparity:
    - Woodford (2003): 'Timeless perspective'
    - Heuristic focus on zero cap. taxes
- Alternatively, could neglect commitment & study Markov policy (Klein et al., 2008, ...) – but surrenders Pareto gains

The rest of the talk

1. Specific example: capital tax problem
2. Time inconsistency and promises
3. Recursively Pareto efficient promise choices
4. Back to the example

Example: Capital taxation

Paper considers general setting, but to fix ideas focus talk on Judd (1985) problem (also SW, 2015):

- **Workers** supply labour **inelastically**, receive wage income
  - Consumption c_t
- **Captalists** own capital stock, do not work
  - Consumption C_t
- **Government** taxes capital & wage income (linearly), fixed expenditure need g
  - Weighted-utilitarian prefs, rel. weight on capitalists μ^c

Example: Capital taxation

Main problem

In primal form, Ramsey problem is:

$$\max_{\{c_t, C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + \mu^c u\left(C_t\right) \right]$$

s.t.

$$c_t + C_t + k_{t+1} + g \leq F\left(k_t\right) + \left(1 - \delta\right) k_t \quad (1)$$

$$u_{C,t} k_{t+1} \leq \beta u_{C,t+1} \left(C_{t+1} + k_{t+2}\right) \quad (2)$$

(& k_0 given)

Promise values

- Constraint (2) implies time inconsistency. Loosely:
  - Promise high $t+1$ wealth to capitalist when k_{t+1} high
  - Ex-post incentive to deviate
- Will deconstruct this in terms of promise values
- Similar to usual APS (1990) objects, but in sequence space: $\{\omega_t\}_{t=0}^{\infty}$

Promise values

- Two promises relevant at t:
  1. Promise made for $t+1$, ω^m_{t+1}:

$$u_{C,t} k_{t+1} \leq \beta \omega^m_{t+1} \tag{3}$$

  2. Promise kept at t, ω^k_t:

$$u_{C,t} \left(C_t + k_{t+1} \right) \geq \omega^k_t \tag{4}$$

- REE $\Rightarrow$ $\omega^m_{t+1} = \omega^k_{t+1} := \omega_{t+1}$ $\Rightarrow$ back to constraint (2)

Inner problem

- Given any pair of sequences $\left\{\omega_t^k, \omega_{t+1}^m\right\}$, I have a well-defined 'inner problem':
- Max of $\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + \mu^c u\left(C_t\right)\right]$ on resources, PK & PM (all t)
- Value: $V\left(\left\{\omega_t^k, \omega_{t+1}^m\right\}_{t=0}^{\infty}, k_0\right)$

Inner problem

Time-consistency

- **Result**: *This 'inner problem' is time-consistent*

$$V\left(\left\{\omega_t^k,\omega_{t+1}^m\right\}_{t=0}^{\infty},k_0\right)=u\left(c_0^*\right)+\mu^c u\left(C_0^*\right)+\beta V\left(\left\{\omega_t^k,\omega_{t+1}^m\right\}_{t=1}^{\infty},k_1^*\right)$$

- Intuition: FL constraints replaced with static restrictions...
- Distills time inconsistency into choice of promises
- Very general construct: works whenever FL constraints additively separable

Outer problem: time-inconsistency

- $V\left(\left\{\omega_t^k, \omega_{t+1}^m\right\}_{t=s}^{\infty}, k_s\right)$ a preference structure over possible promise sequences, parameterised by k_s
- There is time inconsistency in promise choice:

$$\{\omega_t^*\}_{t=0}^{\infty} = \arg \max_{\{\omega_t\}_{t=0}^{\infty}} V\left(\{\omega_t, \omega_{t+1}\}_{t=0}^{\infty}, k_0\right)$$

$$\{\omega_t^*\}_{t=1}^{\infty} \neq \arg \max_{\{\omega_t\}_{t=1}^{\infty}} V\left(\{\omega_t, \omega_{t+1}\}_{t=1}^{\infty}, k_1^*\right)$$

- Could resolve this by choosing a dictator (Ramsey)...
- ... or could ask what Pareto efficiency requires
  - [Standard approach to conflicting wants...]

Pareto efficiency

Definition for choice of promises

- This motivates a definition of Pareto efficiency *in choice of promises*:
- **Definition 1**: $\{\omega_t\}_{t=\tau}^{\infty}$ is *Pareto efficient for period* $\tau \geq 0$ iff there does not exist $\{\omega_t'\}_{t=\tau}^{\infty}$ s.t. for all $s \geq \tau$:

$$V\left(\left\{\omega_t'\right\}_{t=s}^{\infty}, k_s^*\right) - V\left(\{\omega_t\}_{t=s}^{\infty}, k_s^*\right) \geq \varepsilon > 0$$

  - $\{k_t^*\}_{t=\tau}^{\infty}$ is capital stock path when $\{\omega_t\}_{t=\tau}^{\infty}$ chosen
  - ε a scalar (time-invariant)

Pareto efficiency

Some observations

1. Defined as absence of **strict** gains from switching to alternative (sustained at limit)
   - Gives largest choice set possible
2. This alternative is the same (in continuation) for all:
   - $\{\omega'_t\}_{t=\tau}^{\infty}$ for policymaker in τ, $\{\omega'_t\}_{t=\tau+1}^{\infty}$ for policymaker in $\tau + 1$, etc
   - [Policymaker at t always strictly prefers to start new Ramsey plan from t...]

Pareto efficiency

Some observations

3. Clearly Ramsey promise sequence is PE for period 0...
    - ... But continuation may not be for $t > 0$
4. On-path definition, applied to promises only
    - Alternative: PE as 'round table' at start of time – c.f. FW (2007)...
    - Would allow for disagreement over cap accumulation
    - ...Takes us beyond time inconsistency problem
    - Not easy to apply recursively

- Deep connection between our approach & idea of different periods' policymakers 'trading' promises
- 'The more generous is my ω_t^k, the better is the ω_{t+1}^m I can buy'
- [Samuelson OLG!]

Recursive PE

Definition and results

- **Definition 2**: $\{\omega_t\}_{t=0}^{\infty}$ is *recursively Pareto* efficient iff the continuation $\{\omega_t\}_{t=\tau}^{\infty}$ is Pareto efficient for all $\tau \geq 0$

- **Result (a)**: *Recursive PE policy exists (conditions in paper)*

- **Result (b)**: *RPE* $\{\omega_t\}_{t=0}^{\infty}$ *series converge to ss different from Ramsey, if they converge at all*

- Marginal cost of keeping promises is lower under RPE, vs Ramsey

Recursive PE vs Ramsey

- Ramsey: *No alternative promise sequence delivers higher welfare to first policymaker*
- Recursive PE: *No alternative promise sequence that all policymakers from* t *onwards would rather switch to, some* $t \geq 0$

(Getting) Back to the example

A brief cookbook

- How to apply this idea to cap. tax example?
- Recursive PE 'cookbook' works as follows:

1. Solve inner problem, given choice of $\left\{\omega_t^k, \omega_{t+1}^m\right\}$
   - Conditions in terms of multipliers on PK and PM: $\{\lambda_t^k, \lambda_t^m\}$
2. Select $\left\{\omega_t^k, \omega_{t+1}^m\right\}$ consistent with RPE
   - In practice, a restriction on mutlipliers
     - $\lambda_t^k = \beta\lambda_t^m$ (most cases)
   - Will be consistent with stationarity, where Ramsey multipliers are not ($\lambda_t^k = \lambda_{t-1}^m$)
   - RPE alone not enough to pin down transition – one dimension of indeterminacy
   - Resolve by preserving additive separability in choice rules ⇒ simple conditions

Back to the example

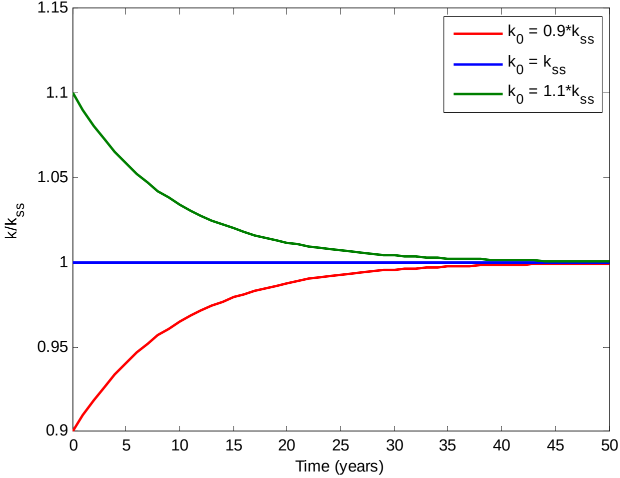
- Recursive PE policies consistent with following policy rule:

$$\underbrace{\left[\beta\eta_{t+1}\left(1+F_{k,t+1}-\delta\right)-\eta_t\right]}_{\text{Capital wedge}}\cdot k_{t+1}=\underbrace{\left[\eta_t-\mu^c u_{C,t}\right]}_{\text{Consumption wedge}}\cdot C_t$$

  - η_t: resource multiplier
- Simple problem, simple rule ... once time-inconsistent dynamics filtered away
- Underlying trade-off: giving capitalists wealth incentivises accumulation, but at cost of consumption
- Next slide: simulation with CES prefs, $\sigma = 2$, $\mu^c = 0$, $\beta = 0.96$

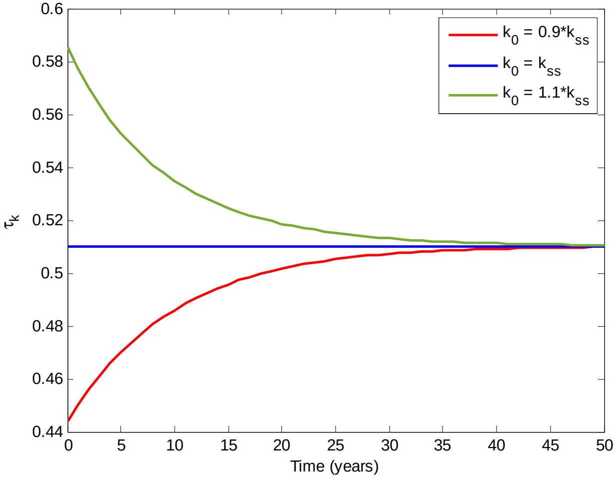
RPE capital transition

A boring transition...



RPE capital taxes

... And a boring path for taxes



Concluding points

- In light of SW (2015), dull policies may be quite interesting!
- Similarly significant implications when RPE applied to other 'problematic' Ramsey outcomes:
  - Simple constant policies in LQ models
  - Stationary cross-sectional P-weights in limited commitment models à la Kocherlakota (1996)
  - Immiseration... (ongoing work)
- Invites reasonable scepticism: *Is it ever appropriate to commit to something that is not Ramsey?*
- If you can be a dictator, should you be?

Example: Insurance with limited commitment

One-sided LC

- μ of population receives stochastic income (high/low)
- $(1-\mu)$ of population permanently low income
- Utilitarian govt provides insurance with redistributive component (no savings)
- Agents can walk away from scheme, eat endowment

Example: Insurance with limited commitment

Ramsey solution

- Ramsey solution well known, easiest to represent with recursive multipliers:

$$u'\left(c_t^i\right) = \frac{\eta_t}{1+\lambda_t^i}$$

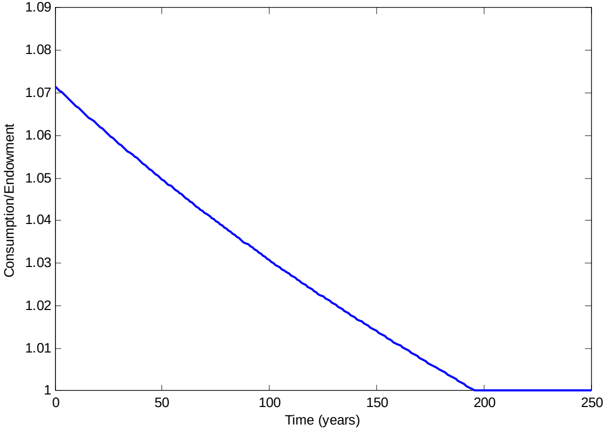
- η_t: resource multiplier
- λ_t^i: sum of multipliers on past PCs:

$$\lambda_t^i = \lambda_{t-1}^i + \nu_t^i$$

- $\nu_t^i > 0 \Leftrightarrow$ PC for i binds at t
- $\lambda_{-1} = 0$
- *Relative* P-weight of low-income types diminishes over time

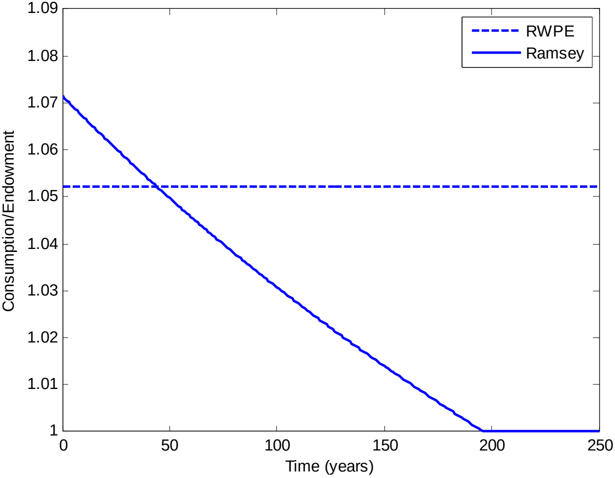
Example: Redistribution with participation constraints

One-sided LC: consumption of types with low income draw



Redistribution problem revisited

Consumption of low earners



Trading promises

A heuristic approach

- Let generation at s face the problem:

$$\max_{\omega_s^k, \omega_{s+1}^m} V\left(\left\{\omega_t^k, \omega_{t+1}^m\right\}_{t=s}^{\infty}, k_s\right)$$

s.t.:

$$p_s\left(\omega_s^k - \bar{\omega}_s\right) \geq p_{s+1}\left(\omega_{s+1}^m - \bar{\omega}_{s+1}\right)$$

(other promises given)

- 'Earn' from keeping promises, 'spend' on making them
- Analogy to textbook Samuelson OLG...
- Equilibrium: $\left\{p_t, \omega_t^k, \omega_{t+1}^m\right\}$ with $\omega_t^k = \omega_t^m, \forall t > 0$, given $\{\bar{\omega}_t\}$