Time-Consistent Institutional Design

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Introduction

- This is a normative paper about Kydland & Prescott (1977) problems
  - Expectations of future policies affect current choice set
  - Benefits to making binding promises, *ex-post* incentives to break them
- Everywhere in macro:
  - Capital taxation, inflation bias, dynamic social insurance, human capital policies, ...
- Our focus is general, but without aggregate risk
What we do
A new way to choose desirable policies (promises)

- Time inconsistency ⇔ no policy is recursively optimal
- Ramsey: impose initial prefs
  - Recursive optimality
  - [Recursive implementation possible, via expanded state space]
- This paper: a desirable policy that remains desirable
  - Recursive optimality
  - Pareto criterion, applied across time
  - Applied exclusively to choice of promises (c.f. Farhi & Werning, 2007)

- Assume policymaker has commitment device – ‘institutional design’
Why do this?

▶ Main motivation: *Ramsey policy does not seem to answer practical policy questions*

▶ Two features commonly viewed as ‘problematic’:
  1. Substantial disparity between long & short run choices

▶ E.g.: Variant of Judd (1985):

![Graphs showing Capital Taxes and Capital normalized to steady state over time](image-url)
Why do this?

- Main motivation: *Ramsey policy does not seem to answer practical policy questions*
- Two features of Ramsey policy often viewed as ‘problematic’:
  1. Substantial disparity between long & short run choices
  2. Long-run outcomes may not be desirable *per se*

  - *Immiseration results* ([Thomas & Worrall, 1990, dynamic Mirrlees, ...])
  - Optimal cap. taxes ⇒ ‘corner’ outcomes ([Straub & Werning, 2015])

- *If I can* find a ‘recursively desirable’ policy, can deal with both
Related work

- Important works responding to these concerns, but quite context-specific:
  - Long run desirability/immiseration:
    - Phelan (2006), Farhi & Werning (2007): change Pareto weighting on different generations
  - Short run/long run disparity:
    - Woodford (2003): ‘Timeless perspective’
    - Heuristic focus on zero cap. taxes

- Alternatively, could neglect commitment & study Markov policy (Klein et al., 2008, ...) – but surrenders Pareto gains
The rest of the talk

1. Specific example: capital tax problem
2. Time inconsistency and promises
3. Recursively Pareto efficient promise choices
4. Back to the example
Example: Capital taxation

Paper considers general setting, but to fix ideas focus talk on Judd (1985) problem (also SW, 2015):

- **Workers** supply labour *inelastically*, receive wage income
  - Consumption $c_t$
- **Capitalists** own capital stock, do not work
  - Consumption $C_t$
- **Government** taxes capital & wage income (linearly), fixed expenditure need $g$
  - Weighted-utilitarian prefs, rel. weight on capitalists $\mu^c$
Example: Capital taxation

Main problem

In primal form, Ramsey problem is:

\[
\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + \mu^c u(C_t)]
\]

s.t.

\[
c_t + C_t + k_{t+1} + g \leq F(k_t) + (1 - \delta) k_t \quad (1)
\]

\[
u_{C,t}k_{t+1} \leq \beta u_{C,t+1} (C_{t+1} + k_{t+2}) \quad (2)
\]

(& \, k_0 \, \text{given})
Promise values

- Constraint (2) implies time inconsistency. Loosely:
  - Promise high $t + 1$ wealth to capitalist when $k_{t+1}$ high
  - Ex-post incentive to deviate
- Will deconstruct this in terms of promise values
- Similar to usual APS (1990) objects, but in sequence space: $\{\omega_t\}_{t=0}^{\infty}$
Promise values

- Two promises relevant at $t$:
  1. Promise made for $t + 1, \omega_{t+1}^m$:

\[
u_{C,t} k_{t+1} \leq \beta \omega_{t+1}^m\]  \hspace{1cm} (3)

  2. Promise kept at $t, \omega_t^k$:

\[
u_{C,t} (C_t + k_{t+1}) \geq \omega_t^k\]  \hspace{1cm} (4)

- $\text{REE} \Rightarrow \omega_{t+1}^m = \omega_{t+1}^k := \omega_{t+1} \Rightarrow \text{back to constraint (2)}$
Given any pair of sequences \( \{ \omega_t^k, \omega_{t+1}^m \} \), I have a well-defined ‘inner problem’:

- Max of \( \sum_{t=0}^{\infty} \beta^t [u(c_t) + \mu^c u(C_t)] \) on resources, PK & PM (all \( t \))
- Value: \( V\left( \{ \omega_t^k, \omega_{t+1}^m \}_{t=0}^{\infty}, k_0 \right) \)
Result: This ‘inner problem’ is time-consistent

\[ V \left( \left\{ \omega^k_t, \omega^m_{t+1} \right\}_{t=0}^{\infty}, k_0 \right) = u(c^*_0) + \mu^c u(C^*_0) + \beta V \left( \left\{ \omega^k_t, \omega^m_{t+1} \right\}_{t=1}^{\infty}, k_1^* \right) \]

Intuition: FL constraints replaced with static restrictions...
Distills time inconsistency into choice of promises
Very general construct: works whenever FL constraints additively separable
Outer problem: time-inconsistency

- $V \left( \{ \omega_t^k, \omega_{t+1}^m \}_{t=s}^{\infty}, k_s \right)$ a preference structure over possible promise sequences, parameterised by $k_s$

- There is time inconsistency in promise choice:

  - $\{ \omega_t^* \}_{t=0}^{\infty} = \arg \max_{\{ \omega_t \}_{t=0}^{\infty}} V (\{ \omega_t, \omega_{t+1} \}_{t=0}^{\infty}, k_0)$
  - $\{ \omega_t^* \}_{t=1}^{\infty} \neq \arg \max_{\{ \omega_t \}_{t=1}^{\infty}} V (\{ \omega_t, \omega_{t+1} \}_{t=1}^{\infty}, k_1^*)$

- Could resolve this by choosing a dictator (Ramsey)...
- ... or could ask what Pareto efficiency requires
  - [Standard approach to conflicting wants...]

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This motivates a definition of Pareto efficiency in choice of promises:

**Definition 1:** \( \{ \omega_t \}_{t=\tau}^\infty \) is Pareto efficient for period \( \tau \geq 0 \) iff there does not exist \( \{ \omega'_t \}_{t=\tau}^\infty \) s.t. for all \( s \geq \tau \):

\[
V ( \{ \omega'_t \}_{t=s}^\infty, k_s^* ) - V ( \{ \omega_t \}_{t=s}^\infty, k_s^* ) \geq \varepsilon > 0
\]

- \( \{ k_t^* \}_{t=\tau}^\infty \) is capital stock path when \( \{ \omega_t \}_{t=\tau}^\infty \) chosen
- \( \varepsilon \) a scalar (time-invariant)
Pareto efficiency

Some observations

1. Defined as absence of strict gains from switching to alternative (sustained at limit)
   - Gives largest choice set possible

2. This alternative is the same (in continuation) for all:
   - $\{\omega'_t\}_{t=\tau}^\infty$ for policymaker in $\tau$, $\{\omega'_t\}_{t=\tau+1}^\infty$ for policymaker in $\tau + 1$, etc
   - [Policymaker at $t$ always strictly prefers to start new Ramsey plan from $t$...]

...
3. Clearly Ramsey promise sequence is PE for period $0$...
   ▶ ... But continuation may not be for $t > 0$

4. On-path definition, applied to promises only
   ▶ Alternative: PE as ‘round table’ at start of time – c.f. FW (2007)...
   ▶ Would allow for disagreement over cap accumulation
   ▶ ...Takes us beyond time inconsistency problem
   ▶ Not easy to apply recursively

▶ Deep connection between our approach & idea of different periods’ policymakers ‘trading’ promises
▶ ‘The more generous is my $\omega^k_t$, the better is the $\omega^m_{t+1}$ I can buy’
▶ [Samuelson OLG!]
Recursive PE
Definition and results

- **Definition 2:** \( \{\omega_t\}_{t=0}^{\infty} \) is *recursively Pareto* efficient iff the continuation \( \{\omega_t\}_{t=\tau}^{\infty} \) is Pareto efficient for all \( \tau \geq 0 \)

- **Result (a):** Recursive PE policy exists (conditions in paper)

- **Result (b):** RPE \( \{\omega_t\}_{t=0}^{\infty} \) series converge to ss different from Ramsey, if they converge at all

- Marginal cost of keeping promises is lower under RPE, vs Ramsey
Recursive PE vs Ramsey

- **Ramsey**: No alternative promise sequence delivers higher welfare to first policymaker.

- **Recursive PE**: No alternative promise sequence that all policymakers from $t$ onwards would rather switch to, some $t \geq 0$. 
(Getting) Back to the example
A brief cookbook

- How to apply this idea to cap. tax example?
- Recursive PE ‘cookbook’ works as follows:

1. Solve inner problem, given choice of \( \{ \omega_k^t, \omega_{t+1}^m \} \)
   - Conditions in terms of multipliers on PK and PM: \( \{ \lambda_k^t, \lambda_m^t \} \)

2. Select \( \{ \omega_k^t, \omega_{t+1}^m \} \) consistent with RPE
   - In practice, a restriction on multipliers
     - \( \lambda_k^t = \beta \lambda_m^t \) (most cases)
     - Will be consistent with stationarity, where Ramsey multipliers are not \( \lambda_k^t = \lambda_{t-1}^m \)
   - RPE alone not enough to pin down transition – one dimension of indeterminacy
   - Resolve by preserving additive separability in choice rules ⇒ simple conditions
Recursive PE policies consistent with following policy rule:

\[
[\beta \eta_{t+1} (1 + F_{k,t+1} - \delta) - \eta_t] \cdot k_{t+1} = [\eta_t - \mu^c u_C,t] \cdot C_t
\]

- Capital wedge
- Consumption wedge

- \(\eta_t\): resource multiplier

Simple problem, simple rule ... once time-inconsistent dynamics filtered away

Underlying trade-off: giving capitalists wealth incentivizes accumulation, but at cost of consumption

Next slide: simulation with CES prefs, \(\sigma = 2, \mu^c = 0, \beta = 0.96\)
RPE capital transition

A boring transition...

\[
\frac{k}{k_{ss}} = 0.9 \times k_{ss} \\
\frac{k}{k_{ss}} = k_{ss} \\
\frac{k}{k_{ss}} = 1.1 \times k_{ss}
\]
RPE capital taxes

... And a boring path for taxes
Concluding points

- In light of SW (2015), dull policies may be quite interesting!
- Similarly significant implications when RPE applied to other ‘problematic’ Ramsey outcomes:
  - Simple constant policies in LQ models
  - Stationary cross-sectional P-weights in limited commitment models à la Kocherlakota (1996)
  - Immiseration... (ongoing work)
- Invites reasonable scepticism: *Is it ever appropriate to commit to something that is not Ramsey?*
- If you can be a dictator, should you be?
Example: Insurance with limited commitment

One-sided LC

- $\mu$ of population receives stochastic income (high/low)
- $(1 - \mu)$ of population permanently low income
- Utilitarian govt provides insurance with redistributive component (no savings)
- Agents can walk away from scheme, eat endowment
Example: Insurance with limited commitment

Ramsey solution

- Ramsey solution well known, easiest to represent with recursive multipliers:
  \[ u' \left( c_t^i \right) = \frac{\eta_t}{1 + \lambda_t^i} \]

- \( \eta_t \): resource multiplier
- \( \lambda_t^i \): sum of multipliers on past PCs:
  \[ \lambda_t^i = \lambda_{t-1}^i + \nu_t^i \]

- \( \nu_t^i > 0 \) ⇔ PC for \( i \) binds at \( t \)
- \( \lambda_{-1} = 0 \)
- *Relative* P-weight of low-income types diminishes over time
Example: Redistribution with participation constraints

One-sided LC: consumption of types with low income draw
Redistribution problem revisited

Consumption of low earners

![Graph showing the consumption/ endowment over time for RWPE and Ramsey solutions. The graph illustrates the trend of consumption relative to endowment over varying time periods, highlighting the differences between the two approaches. The x-axis represents time in years, ranging from 0 to 250, while the y-axis indicates the consumption/endowment ratio, showing a gradual decrease over time for the Ramsey solution and a more stable line for the RWPE solution.]
Trading promises
A heuristic approach

Let generation at $s$ face the problem:

\[
\max_{\omega^k_s, \omega^m_{s+1}} V \left( \left\{ \omega^k_t, \omega^m_{t+1} \right\}_{t=s}^{\infty}, k_s \right)
\]

s.t.:

\[
p_s \left( \omega^k_s - \bar{\omega}_s \right) \geq p_{s+1} \left( \omega^m_{s+1} - \bar{\omega}_{s+1} \right)
\]

(other promises given)

‘Earn’ from keeping promises, ‘spend’ on making them

Analogy to textbook Samuelson OLG...

Equilibrium: \( \{ p_t, \omega^k_t, \omega^m_{t+1} \} \) with \( \omega^k_t = \omega^m_t, \forall t > 0 \), given \( \{ \bar{\omega}_t \} \)