

# Self-Fulfilling Recessions at the Zero Lower Bound\*

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**Abstract:** We draw attention to an overlooked channel through which purely self-fulfilling recessions can occur in monetary economies subject to a zero bound on nominal interest rates. In many environments depressed contemporary economic conditions imply lower expectations of future output and inflation. We show that if this expectations channel is sufficiently strong, a current recession followed by gradual convergence back to steady state may be a self-supporting equilibrium outcome. We present the logic behind this result heuristically in partial equilibrium, and in a nonlinear New Keynesian model, and explain why it is different from the well-known multiplicity highlighted by Benhabib, Schmitt-Grohé and Uribe (2001). Generically, multiplicity of the kind we identify occurs only in certain regions of the parameter space. We study two well-known estimated DSGE models to investigate how likely it is that multiplicity could arise. We find that purely self-fulfilling recessions are possible at 99.8 per cent of parameter draws from the posterior distribution in the Smets and Wouters (2007) model, and at over two-thirds of draws in the Iacoviello and Neri (2010) model. Alternative monetary policy strategies can play an important role in reducing these probabilities.

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\*The analysis and conclusions set forth in this paper are those of the authors and do not indicate concurrence by other members of the research staff or the Federal Reserve Board of Governors. We thank seminar participants at Cambridge and Surrey for helpful feedback. All errors are ours.

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# 1 Introduction

The precise implications of the zero lower bound (ZLB) constraint for macroeconomic dynamics has been the subject of substantial research attention in recent years, for obvious reasons. Within the resulting literature, an important analytical distinction has emerged between papers that assume a unique equilibrium dynamic, and those whose focus is on multiplicity and coordination traps.<sup>1</sup> The unique equilibrium approach generally follows the influential early contribution of Eggertsson and Woodford (2003), allowing for some fundamental shock to drive the economy's 'natural' rate of interest below the zero bound. After a sufficiently long time horizon this shock dissipates, and normality resumes, with positive nominal rates.<sup>2</sup> The multiplicity approach emphasises the inability of central banks to rule out certain deflationary traps when there is a limit to how far nominal rates can be cut.<sup>3</sup>

The present paper is in the multiplicity tradition. Our main contribution is to identify and characterise a novel form of equilibrium multiplicity that can arise across a large class of well-known dynamic general equilibrium models in the presence of the ZLB, and can account for deep, self-sustaining recessions. The issue occurs whenever there is a sufficiently strong structural link between depressed contemporary economic conditions and low future output and inflation. When this link is present, a contraction in output in the current period will have a direct negative impact on future economic conditions, and on current expectations of future economic conditions. If there is a limit to how far nominal interest rates can be cut, the anticipation of future stagnation, together with low or negative inflation, may be enough to support low current output as an equilibrium outcome.

This dynamic is reminiscent of, but distinct from, the well-known deflationary traps highlighted by Benhabib, Schmitt-Grohé and Uribe (2001). In these authors' work, 'pessimistic' private-sector expectations play a direct causal role: if agents anticipate low enough future inflation and output, and if the nominal interest rate cannot fall below zero, the expected real interest rate may be stuck inefficiently high, consistent with low current demand and production. So long as the state of pessimism persists, low future output and inflation – as anticipated – will likewise follow. A key modelling input in papers that follow this approach is thus the state of economic 'sentiment' – the degree of pessimism or optimism with which agents view any given set of economic fundamentals. Sentiment is explicitly treated

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<sup>1</sup>We use the terms 'multiplicity' and 'coordination trap' interchangeably.

<sup>2</sup>This is by far the more common approach. Recent influential examples include Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011), Werning (2012) and Wieland (2014). Current work by Cochrane (2015) questions the equilibrium selection criterion that these papers use.

<sup>3</sup>This work generally builds on the influential contribution of Benhabib, Schmitt-Grohé and Uribe (2001). Important recent examples include Mertens and Ravn (2014), Aruoba and Schorfheide (2013) and Schmitt-Grohé and Uribe (2013).

as an exogenous, persistent random variable by Mertens and Ravn (2014) and Aruoba and Schorfheide (2013).<sup>4</sup>

The main difference between our work and these analyses is that we allow pessimism about the future to be driven by observed changes in endogenous economic variables. It is not that a bad state of sentiment leaves consumers more pessimistic for any given set of fundamentals. Instead, the fundamentals themselves weaken to an extent that justifies pessimism.

More formally, the distinction is easiest to understand by reference to the idea of an expectations mapping. Outcomes in forward-looking economic environments depend, by definition, upon the expectations of some vector of variables, say  $\mathbb{E}_t x_{t+1}$ . To determine equilibrium in period  $t$ , a relationship needs to be specified that fixes these expectations in terms of  $t$ -dated variables – of the form  $\mathbb{E}_t x_{t+1} = f(x_t)$ , for some ‘expectations mapping’  $f(\cdot)$ . For expectations to be rational, this mapping must itself be model-consistent, though in principle it is possible to analyse equilibrium possibilities for any arbitrary  $f(\cdot)$ .

The deflation trap and indeterminacy highlighted by Benhabib et al. (2001) comes from the fact that many possible choices for the function  $f(\cdot)$  may be consistent with rational expectations equilibrium. A deflation trap occurs when a ‘pessimistic’  $f(\cdot)$  has been selected. Accordingly, the sentiment shocks analysed in Mertens and Ravn (2014) and Aruoba and Schorfheide (2013) effectively deliver fluctuations by changing the  $f(\cdot)$  function that is being used. This makes the expectations-formation process central to whether such deflationary traps are a genuine risk. Work on adaptive learning rules by Evans et al. (2008) and Benhabib et al. (2014) has thus been concerned with the process by which convergence on a particular rational expectations  $f(\cdot)$  function is reached. In general the results are positive: least-squares learning delivers convergence to the unique desirable steady-state  $f(\cdot)$  mapping, not the Benhabib et al. (2001) deflationary outcome, provided the usual Taylor principle is satisfied.<sup>5</sup> This has often been used as a justification for focusing on fluctuations in the neighbourhood of the desirable steady state.<sup>6</sup>

The multiplicity that we highlight does not depend on the existence of multiple  $f(\cdot)$  functions, nor on the selection of a particularly ‘pessimistic’  $f(\cdot)$ . Even when the expectations mapping is fixed, and is consistent with desirable steady-state outcomes, we show that there may be at least two rational expectations equilibria. One is benign, but the other involves a collapse in current output, low future inflation, and nominal interest rates constrained at zero – the dynamic described above. Low inflation expectations and a high real interest

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<sup>4</sup>The original work by Benhabib et al. (2001) used a perfect-foresight approach, equivalent to the state of pessimism persisting with probability one.

<sup>5</sup>The exception is when negative shocks to inflation are particularly severe. In this case an extreme deflationary spiral can emerge, but the dynamics are not convergent.

<sup>6</sup>See, for instance, McCallum (2003).

rate are the effect of the particular realisation of  $x_t$ , passed through the given function  $f(\cdot)$ . This means our multiplicity endures even in the event that learning dynamics have induced convergence to the unique desirable  $f(\cdot)$  function.

The bulk of our paper is concerned with exploring the conditions under which this form of multiplicity may arise, and the equilibrium dynamics that it implies. As with many analyses of outcomes at the zero bound, the main intuition can be communicated by reference to the consumption Euler equation. This is the purpose of Section 2, which presents a stylised within-period equilibrium problem, subject to a given – not necessarily rational – expectations mapping. With sufficient persistence embedded in the expectations process, we show that exactly two levels of output are consistent with equilibrium. Either output is high, and expectations of future inflation and production remain high, or output falls, and expectations deteriorate with it. Importantly, the severity of self-fulfilling recessions is greatest when persistence is just marginally above the required threshold. We explain why this is the case, and discuss the implications for the relevance of our mechanism.

Section 3 then provides a more formal discussion of the difference between expectation traps that derive from multiplicity in the set of equilibrium rational expectations mappings, and expectation traps that exist even when the expectations mapping is fixed. As we explain, one of the reasons why our multiplicity problem may not have received much attention in the literature to date is that it requires a non-linear model. In this it contrasts, for instance, with price-level multiplicity in models with interest-rate rules, as analysed by Cochrane (2011).

Having established the foundations for our multiplicity problem, Section 4 provides a simulated example, in which self-fulfilling recessions can arise in a fully-specified, recursive rational expectations equilibrium. The setting is a nonlinear version of the canonical dynamic New Keynesian model, where the only departure from convention is to assume that the monetary policymaker feeds back aggressively on real GDP *growth*, in addition to inflation.<sup>7</sup> This is the most direct way that we have found to impart the necessary persistence into the well-studied dynamic New Keynesian setting. If output collapses in the present period, reversion back to steady state means that it will be expected to grow in the future. Because of the feedback rule, this implies that future monetary policy can be expected to be relatively tight, and this in turn ensures that the future *level* of output and the future inflation rate will be low. Of relevance for policy purposes, we also show that conditional on a recession occurring, marginal increases in government expenditure during the zero bound episode are contractionary. Yet we argue that this cannot be interpreted simply as a ‘negative multiplier’ result. This is because sufficiently large counter-cyclical expenditure can be enough to rule out multiplicity altogether. Thus the case for fiscal intervention can be made,

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<sup>7</sup>The Taylor principle remains satisfied.

provided it is large enough.

For any given macroeconomic model, our coordination traps will usually exist in some regions of the parameter space but not others, as they rely on a sufficiently strong structural link existing between current and future economic conditions. This makes their empirical relevance an important open question. To test it, in Section 5 we take two prominent medium-scale DSGE models ‘off the shelf’, and investigate the likelihood that their parameters will be consistent with multiplicity when a zero bound constraint is added to them. The models are those due to Smets and Wouters (2007) and Iacoviello and Neri (2010), both estimated by Bayesian techniques on pre-crisis US data. We find multiplicity at 99.8 per cent of draws from the posterior parameter distribution for the Smets-Wouters model, and 69 per cent in the Iacoviello-Neri model. We infer from this that the parametric restrictions sufficient for coordination traps are not particularly strong, given the extent of persistence that is hard-wired into such models.

Importantly, however, this result exhibits strong dependence on the exact specification of monetary policy. Mirroring the simulated example of Section 4, an interest rate rule that feeds back on the growth rate of GDP makes multiplicity far more likely. This sort of feedback is assumed in the benchmark specifications of both models. When it is removed, and replaced instead by feedback on the output gap, the posterior likelihood attached to the multiplicity region falls to 61 per cent in the Smets-Wouters model, and just 5.6 per cent in the Iacoviello-Neri model. We take from this a further policy lesson: monetary policy should be discouraged from feeding back on the growth rates of variables.

## 2 Partial equilibrium intuition

The main intuition behind our results can be seen by considering a partial equilibrium environment, focusing a single time period. There is a representative consumer making a consumption-savings decision within this period, and the consumer’s expectations over future variables of interest are summarised by a known mapping from current equilibrium outcomes. We do not specify the equilibrium problem in future time periods that gives rise to this expectations mapping, and need not even take a stance on whether the expectations are rational in the usual sense. This is why we describe the equilibrium as partial. As emphasised in the introduction, a central conceptual difference between the multiplicity we identify and the well-known deflation trap analysed by Benhabib et al. (2001) is that our dynamic *can* occur for a fixed expectations mapping. We do not obtain multiplicity by changing the expectations mapping.<sup>8</sup>

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<sup>8</sup>Section 3 explains the difference between the two approaches in more detail.

## 2.1 Basic setup

We consider an economy that exists in some time period  $t$ . A representative consumer will receive real income  $Y_t$  in this period, equal to aggregate real output in the economy. This income must be allocated between consumption,  $C_t$ , and savings in a risk-free one-period nominal bond,  $B_t$ . The real price of the bond in terms of the consumption good is  $\frac{1}{P_t}$ , and it will return  $(1 + i_t)$  units of the equivalent bond at  $t + 1$  for every unit invested. The consumer's budget constraint in  $t$  is:

$$\frac{1}{P_t} (1 + i_{t-1}) B_{t-1} + Y_t \geq C_t + \frac{1}{P_t} B_t \quad (1)$$

where the first term on the left-hand side captures initial asset holdings. We assume that there is no government, so net asset positions will in fact be zero in any equilibrium – hence we can set  $B_{t-1} = 0$ .

Savings decisions in  $t$  are taken to maximise the utility criterion:

$$\frac{C_t^{1-\sigma} - 1}{1-\sigma} + \beta \hat{\mathbb{E}}_t V \left( \frac{1}{P_{t+1}} (1 + i_t) B_t \right) \quad (2)$$

where the value function  $V$  summarises expected continuation utility for any given real income carried into  $t + 1$ , and  $\hat{\mathbb{E}}_t$  is the consumer's subjective period- $t$  expectations operator. The value function is assumed to have the usual recursive definition, so that optimal choice implies the Euler equation:

$$\frac{1}{P_t} C_t^{-\sigma} = \beta (1 + i_t) \hat{\mathbb{E}}_t \frac{1}{P_{t+1}} C_{t+1}^{-\sigma} \quad (3)$$

Since the saving decision is the only choice made by consumers in  $t$ , equation (3) is sufficient for optimality.

Consumption is the only source of demand in the economy, so that market clearing in  $t$  requires:

$$C_t = Y_t \quad (4)$$

Individuals are aware that market clearing must hold in  $t + 1$ , so the Euler equation will hold in equilibrium if and only if the following is true:

$$Y_t^{-\sigma} = \beta (1 + i_t) \hat{\mathbb{E}}_t \Pi_{t+1}^{-1} Y_{t+1}^{-\sigma} \quad (5)$$

where  $\Pi_{t+1}$  denotes the gross rate of inflation at  $t + 1$ .

We do not model the production side of the economy explicitly, except to assume that a

range of positive values of  $Y_t$  is possible in equilibrium, including a ‘full capacity’ level  $\bar{Y}$ .<sup>9</sup> A supply relationship may imply  $P_t$  – or equivalently  $\Pi_t$  – varies with  $Y_t$ , but we set up the model so that neither  $P_t$  nor  $\Pi_t$  will be a relevant statistic for determining within-period equilibrium real outcomes at  $t$ , so this relationship can be ignored.<sup>10</sup>

The central bank targets  $\bar{Y}$  as its desired output level, according to the following interest rate feedback rule:

$$(1 + i_t) = \max \left\{ \bar{R}\Pi^* \left( \frac{Y_t}{\bar{Y}} \right)^\alpha, 1 \right\} \quad (6)$$

for some  $\alpha > 0$ .  $\bar{R}\Pi^* > 1$  is interpreted as a ‘neutral’ nominal interest rate, decomposed into a natural real interest rate  $\bar{R}$  and an inflation target  $\Pi^*$ .<sup>11</sup>

## 2.2 Expectations and temporary equilibrium

We replace the expectations term in (5) with an expectations mapping, linking observed outcomes in  $t$  to expected outcomes in  $t + 1$ . This mapping captures consumers’ perceived relationship between current and future economic conditions. It is entirely possible to generate this mapping endogenously in a manner that is consistent with the usual rational expectations hypothesis, and in the bulk of the paper we will do precisely this. For illustrative purposes, however, here we will simply assume that inflation and the output gap at  $t + 1$  are predicted by a lognormal model linking these variables to the realised value of the output gap in  $t$ . Using lower-case letters to denote natural logarithms, this model is:

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} \sim N \left( \begin{bmatrix} \bar{y} + \rho (y_t - \bar{y}) \\ \pi^* + \delta (y_t - \bar{y}) \end{bmatrix}, \Sigma \right) \quad (7)$$

where  $\rho$ ,  $\delta$  and  $\pi^*$  are coefficients to be estimated, and  $\Sigma$  is the variance-covariance matrix.

There are two interpretations one can put on this system of expectations. One is to view the model in (7) as a simple non-rational belief structure that consumers in  $t$  happen to have about the more complex process determining outcomes in  $t + 1$ . The other is to note that (7) may be the exact equilibrium belief system that emerges in a rational expectations system when the supply block is modelled in a certain way – recalling that at present the supply block is unmodelled. We do not have a preference between these interpretations. The aim is just to focus attention on the within-period equilibrium problem in isolation from the

<sup>9</sup>Clearly in practice potential output will not be time-invariant, but subscripts are omitted to ease notation.

<sup>10</sup>Clearly  $P_t$  features in the expected inflation term in (5), but we will follow convention in assuming this is homogenous of degree zero in the price level. The commonly-assumed ‘Taylor rule’ link from current inflation to the nominal interest rate in the policy rule is not present.

<sup>11</sup>The rule in (6) does not satisfy the Taylor principle, so would not be consistent with a unique non-explosive price path even in the absence of the zero bound. This may matter for the problem of determining an equilibrium expectations mapping, but is not an important concern once this mapping has been specified, as we do explicitly in the next sub-section.

expectations-formation process. In the terminology of Hicks (1939), we are looking for a ‘temporary equilibrium’ in period  $t$ .<sup>12</sup>

The lognormality assumption in (7) is useful, as it allows us to write the implied expectations mapping analytically. It will satisfy:

$$\hat{\mathbb{E}}_t \Pi_{t+1}^{-1} Y_{t+1}^{-\sigma} = \Xi Y_t^{-(\sigma\rho+\delta)} \quad (8)$$

where  $\Xi$  is a composite constant term.<sup>13</sup> The implication is that the Euler condition now requires:

$$Y_t^{-\sigma} = \beta (1 + i_t) \Xi Y_t^{-(\sigma\rho+\delta)} \quad (9)$$

## 2.3 Multiplicity

Our central point is that for plausible parameter values the zero bound can have an important impact on the set of possible equilibria in period  $t$ . The issue hinges on whether a fall in current income raises or reduces individuals’ relative willingness to consume, particularly given the impact that lower  $Y_t$  has on expected future outcomes. To develop the analysis, first notice from (6) that there exists a minimum level of  $Y_t$ , say  $\tilde{Y}$ , that is consistent with  $i_t > 0$ . This is defined by:

$$\tilde{Y} := \bar{Y} (\bar{R}\Pi^*)^{-\frac{1}{\alpha}} < \bar{Y} \quad (10)$$

Given this, the equilibrium condition (9) can be satisfied either at values for  $Y_t > \tilde{Y}$  such that:

$$\Delta Y_t^{-\alpha-\sigma(1-\rho)+\delta} = 1 \quad (11)$$

or at values for  $Y_t \leq \tilde{Y}$  such that:

$$\bar{\Delta} Y_t^{-\sigma(1-\rho)+\delta} = 1 \quad (12)$$

where  $\Delta$  and  $\bar{\Delta}$  are composite constant terms.<sup>14</sup>

The objects on the left-hand sides of (11) and (12) give the representative consumer’s

<sup>12</sup>Woodford (2013) has drawn renewed attention to the idea of ‘temporary equilibrium’ in macroeconomics – that is, a within-period equilibrium, holding fixed a particular specification of the expectations mapping. Grandmont (1977, 1988) developed the concept in detail in a formal Walrasian general equilibrium setting with money.

<sup>13</sup>Defining  $\sigma_{yy}$ ,  $\sigma_{\pi\pi}$  and  $\sigma_{y\pi}$  as the on- and off-diagonal entries in  $\Sigma$ , we have:

$$\Xi_t := \Pi^{*-1} \bar{Y}^{-\sigma(1-\rho)+\delta} \exp \left\{ \frac{1}{2} \left[ \sigma^2 \sigma_{yy} + \sigma_{\pi\pi} + 2\sigma\sigma_{y\pi} \right] \right\}$$

<sup>14</sup>Specifically:

$$\begin{aligned} \Delta &: = [\beta \bar{R}\Pi^* \bar{Y}^{-\alpha} \Xi]^{-1} \\ \bar{\Delta} &: = [\beta \Xi]^{-1} \end{aligned}$$



Parameter	Value	Justification
$\sigma$	2	EIS = 0.5
$\alpha$	0.5	Taylor output feedback
$\rho$	0.91	Stock & Watson
$\delta$	0.425	Stock & Watson

Table 1: Assumed parameter values

marginal rate of substitution between a unit of consumption in period  $t$  and a unit of real savings in period  $t$ , at any given  $Y_t$ .<sup>15</sup> Since  $Y_t = C_t$ , this is equivalent to an inverse demand curve for period- $t$  consumption, allowing for two general equilibrium effects: the endogenous response of nominal rates to output, and the effect of current output on expected future consumption and inflation. The price of period- $t$  consumption in terms of savings is always one for the consumer,<sup>16</sup> which is equivalent to ensuring a perfectly elastic supply curve at a price of one. The equations thus state that temporary equilibrium is reached wherever the demand for current consumption meets this supply curve.

By construction we will ensure that the expectations mapping and feedback rule are together consistent with one ‘desirable’ equilibrium such that  $Y_t = \bar{Y}$ . In this case the policy rule implies  $(1 + i_t) = \bar{R}\Pi^*$ , and from (7) consumers’ point estimates for log output and inflation at  $t + 1$  are then consistent with full capacity and the inflation target being met. It is easy to check that for  $Y_t = \bar{Y}$  to be an equilibrium in  $t$ , we will then need the ‘neutral’ rate  $\bar{R}$  to satisfy:

$$\bar{R} = \beta^{-1} \exp \left\{ -\frac{1}{2} \left[ \sigma^2 \sigma_{yy} + \sigma_{\pi\pi} + 2\sigma\sigma_{y\pi} \right] \right\} \quad (13)$$

In stationary perfect-foresight models the natural real rate is  $\beta^{-1}$ . The present case is almost identical, except that a correction is needed for the fact that consumer uncertainty over outcomes at  $t + 1$  incentivises higher savings.

We are interested in whether other equilibria are possible in addition to this one. This depends on the shape of the inverse demand function for plausible parameter assumptions: does it have just one intersection with the supply curve, or more? To investigate this we fix standard values for the parameters in (11) and (12), which are reported in Table 1. Central to the exercise are the values assumed for the main parameters in (7),  $\rho$  and  $\delta$ . We take these from the detailed US quarterly business cycle statistics reported in Stock and Watson (1999). Notice that there is both a substantial degree of autocorrelation in the log output gap, captured by  $\rho$ , and a strong observed link between the output gap and subsequent inflation, captured by  $\delta$ .

<sup>15</sup>Notice that when  $Y_t = \tilde{Y}_t$  the two expressions coincide.

<sup>16</sup>The within-period budget constraint is  $Y_t \geq C_t + S_t$ , where  $S_t$  are savings.

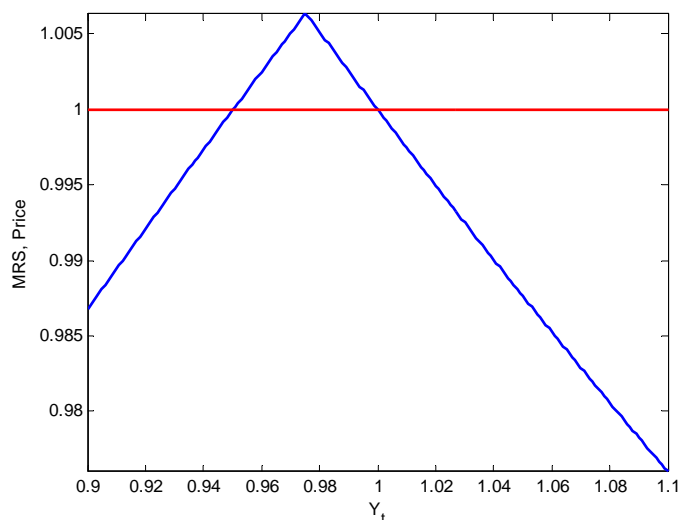


Figure 1: Equilibrium multiplicity in period  $t$

In Figure 1 we graph the system described by equations (11) and (12), given these parameters. Equilibrium obtains where the blue ‘demand’ schedule intersects the horizontal red ‘supply’ schedule. There are exactly two equilibria. The first is at  $Y_t = \bar{Y}$ , as constructed, with  $\bar{Y}$  normalised to 1. The second is an ‘undesirable’ equilibrium, with  $Y_t$  around 5 per cent below  $\bar{Y}$ . This outcome is accompanied by a binding zero bound on the nominal interest rate.

To understand why this multiplicity occurs, consider the different factors that contribute to the relative demand for consumption versus savings in our model economy. There are three relevant channels. The first is a conventional diminishing marginal utility effect. As  $Y_t$  falls,  $C_t$  falls, and this raises the relative benefits to consumption versus savings in period  $t$ . This effect is captured by the  $-\sigma$  term in the exponents on  $Y_t$  in (11) and (12). In isolation, it would imply a monotonically downward-sloping demand curve in Figure 1.

The second channel is a policy effect. So long as  $Y_t > \bar{Y}$ , as  $Y_t$  falls the central bank tends to cut nominal interest rates, and this reduces the relative marginal benefit from saving – thus raising the relative demand for current consumption. This also counts in favour of a downward-sloping demand schedule, and is captured by the  $\alpha$  term in the exponent on  $Y_t$  in (11). Yet this effect will not operate for further falls in  $Y_t$  below  $\bar{Y}$ .

The third channel is an expectations effect. As  $Y_t$  falls the consumer’s expectations of output and inflation at  $t + 1$  also fall, consistent with the model in (7). A reduction in the anticipated value of  $Y_{t+1}$  reduces the anticipated value of  $C_{t+1}$ , and thus raises the expected relative marginal benefits from saving. A reduction in the anticipated value of  $\Pi_{t+1}$  raises the real return from saving (given  $i_t$ ), and this likewise raises the relative benefits from saving. The combination of these two effects causes the relative marginal benefits from con-

sumption in  $t$  to fall, and so *ceteris paribus* it will tend to cause the demand curve in Figure 1 to slope upwards. This expectations channel is captured by the term  $\sigma\rho + \delta$  in the exponents in (11) and (12).

So long as  $Y_t > \tilde{Y}$  the zero bound does not bind, and the marginal utility and policy channels dominate. Lower values for  $Y_t$  are associated with greater relative demand for current consumption. This follows from the fact that the overall exponent on  $Y_t$  in (11) is negative. But once the zero bound binds, further falls in  $Y_t$  reduce rather than increase the relative demand for current consumption. This occurs because the marginal utility effect by itself is dominated by the expectations effect. As incomes fall in period  $t$ , consumers become more pessimistic about their future consumption levels, and anticipate a lower level of future inflation. Together these expectations are enough to increase the relative value that they place on saving at the margin, even though the marginal value of current consumption is simultaneously increasing. The undesirable equilibrium represents a situation where output has fallen enough for consumers to be content with their lower consumption levels, given the pessimism that low output has itself induced.

## 2.4 Persistence and the severity of expectations traps

The general lesson that we take from this is that persistence and propagation in macroeconomic dynamics can enable self-fulfilling recessions to occur at the zero bound. Persistence in output means that current recessions raise the expected future marginal utility of consumption, and a propagation mechanism from low current output to low future inflation raises the real returns from saving. Together these mean that low-income, low-consumption outcomes are equilibrium-consistent. This is a general channel that has been overlooked in the literature to date, and the remainder of our paper is concerned with understanding its implications and scope in more detail.

An important further observation that can be made in the partial equilibrium setting is that the presence of multiplicity depends on assumed parameter values. In this particular case, we need that the demand schedule in Figure 1 is upward-sloping for  $Y_t < \tilde{Y}$ , and downward-sloping for  $Y_t > \tilde{Y}$ . This implies the following two restrictions:

$$-\alpha - \sigma(1 - \rho) + \delta < 0 \quad (14)$$

$$-\sigma(1 - \rho) + \delta > 0 \quad (15)$$

The first of these will go through so long as the policy feedback parameter is large enough, and in most environments its equivalent will be satisfied.<sup>17</sup> More important is condition

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<sup>17</sup>Note that a supply-side link from low output to low inflation, together with inflation feedback in the policy rule, would raise the degree of feedback further. In this respect  $\alpha = 0.5$  can be viewed as being on the low

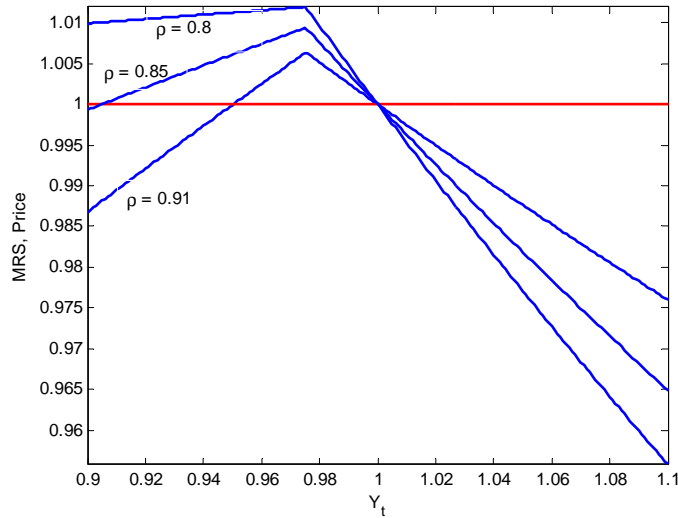


Figure 2: The effects of lower output persistence on equilibrium possibilities

(15), which is a sufficient requirement for the expectations effect to dominate the marginal utility effect when current income falls. As the discussion anticipates, higher values for  $\rho$  and  $\delta$  strengthen the impact of a current recession on expectations, raising the likelihood that a self-fulfilling recession will be possible. By contrast, higher values for  $\sigma$  make the condition harder to meet, at least in the usual case that  $\rho < 1$ . This is because the self-fulfilling dynamic relies in part on an intertemporal substitution channel. Lower expected inflation raises the real interest rate in  $t$ , and this increases the relative incentives to save. The effect on the actual marginal willingness to save depends on the elasticity of intertemporal substitution,  $\frac{1}{\sigma}$ . Higher values of  $\sigma$  imply less of a response to a given real interest rate change, and multiplicity is less likely.

Figure 2 illustrates the effect on the two equilibrium outcomes of reducing the output persistence parameter  $\rho$ , holding all others constant.<sup>18</sup> Though this reduces the strength of the expectations channel, the implication is that the upward-sloping segment of the demand schedule becomes flatter, and this has the effect of worsening the undesirable equilibrium outcome. As the threshold in (15) is approached, extremely severe recessions become admissible. Intuitively, if the expectations channel is only slightly stronger than the marginal utility channel, it takes a very large recession for the relative marginal benefits of current consumption to fall by enough to offset the initial cut in nominal interest rates to zero. Once the threshold in (15) is passed, only the desirable equilibrium remains.

This analysis is important when understanding the implications of policy measures for ameliorating self-fulfilling recessions. In Section 4 we will study a standard fiscal expan-

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side.

<sup>18</sup>Similar qualitative implications follow from reducing  $\delta$  or increasing  $\sigma$ .

sion at the zero bound that initially appears to be very contractionary, conditional on the undesirable equilibrium outcome being realised. For a large enough expansion, however, the undesirable equilibrium ceases to exist. Interpreting the merits of fiscal policy in this context will require a more nuanced perspective than headline multipliers would give.

### 3 (Why) Is this different from BSU (2001)?

It is well known that multiple non-explosive equilibria exist in dynamic rational expectations models with interest-rate rules once the zero bound is incorporated. In this section we explain how the multiplicity problem that we explore differs from this.

The ‘non-fundamental’ liquidity trap was first highlighted by Benhabib, Schmitt-Grohé and Uribe (2001, hereafter BSU). The mechanism is easiest to understand by reference to an efficient Walrasian economy in which only the dynamics of the price level and nominal interest rate remain to be determined. Real income and consumption are constant over time, so the Euler equation for the choice of nominal bonds reduces to:

$$1 = \beta (1 + i_t) \mathbb{E}_t \Pi_{t+1}^{-1} \quad (16)$$

The central bank is choosing an interest-rate rule that satisfies the usual Taylor principle, subject to the zero bound:

$$(1 + i_t) = \max \left\{ \beta^{-1} \Pi^* \left( \frac{\Pi_t}{\Pi^*} \right)^\alpha, 1 \right\} \quad (17)$$

with  $\alpha > 1$ . Any path for inflation and the nominal interest rate that satisfies (16) and (17) in all time periods is consistent with equilibrium.

In general there is a continuum of possible perfect-foresight equilibrium  $\{\Pi_t, i_t\}_{t \geq 0}$  paths. To see this, note that any initial  $\Pi_0$  implies a value for  $i_0$  via (17). This implies a value for  $\Pi_1$  from (16), implying a value for  $i_1$  via (17), and so on. Thus for each initial choice  $\Pi_0 > 0$ , there is a distinct equilibrium. Adding sunspots would expand the equilibrium set still further. Yet the Taylor principle  $\alpha > 1$  means that in the absence of the zero bound all but one of these paths would deliver explosive inflation (or deflation) rates as time progresses, and this has commonly been used as justification for focusing on the unique non-explosive path.<sup>19</sup> This is where  $\Pi_t = \Pi^*$  and  $(1 + i_t) = \beta^{-1} \Pi^*$  for all  $t$ .

The point made by BSU is that when a zero bound is present, there are multiple non-explosive equilibrium price paths. In particular, there are two possible steady-state inflation

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<sup>19</sup>The justification for doing so remains debated. Cochrane (2011) argues strongly that non-explosiveness should not be a selection criterion *per se*.

rates. One is  $\Pi_t = \Pi^*$  for all  $t$  as before; the other is  $\Pi_t = \beta$  for all  $t$ , with  $i_t = 0$ . A continuum of perfect-foresight price paths exists that converges to this latter steady state.

In the efficient Walrasian setting this multiplicity will not affect welfare, but in a New Keynesian environment the low-inflation steady state is associated with steady-state output that is inefficiently low. A liquidity trap of this form has commonly been suggested as an explanation for Japan's long period of stagnation since the 1990s, and more recently as an account of the sustained zero bound episode experienced by most major OECD economies since the Great Recession. The papers by Mertens and Ravn (2014), Aruoba and Schorfheide (2013) and Schmitt-Grohé and Uribe (2013) are examples of this approach.

The difference between BSU and our analysis amounts to a difference between multiplicity in the set of possible expectations mappings (BSU), and multiple temporary equilibria for a given expectations mapping (our paper). In terms of outcomes, this translates into a difference between persistent or permanent self-fulfilling convergence in economic outcomes on an undesirable steady state (BSU), as against a temporary recession in the neighbourhood of a single, fixed steady state (our paper).

By analogy with the model in Section 2, we can consider alternative functional mappings to replace the expectational term  $\mathbb{E}_t \Pi_{t+1}^{-1}$  in (16), in this case explicitly restricting attention to those that are consistent with rational expectations. In principle this mapping could be time-varying, and could depend on a range of sunspot variables realised in period  $t$ . This would allow, for instance, for perfect-foresight paths that converge gradually on the deflationary steady state without immediately jumping to it. But the central point can be made by focusing on simpler mappings that are time-invariant, and depend only on the economy's underlying state vector. Since the model at hand does not have any state variables, this means selecting a function that is in fact a constant. We will denote this constant  $\phi$ , and study equilibria where  $\mathbb{E}_t \Pi_{t+1}^{-1} = \phi$  for all  $t$ .

There are two such mappings that are consistent with rational expectations. One is  $\phi = \Pi^{*-1}$ , and the other is  $\phi = \beta^{-1}$ . As one would expect, these correspond directly to the two possible steady-state inflation rates. These alternative constant predictions are direct analogues to the lognormal predictive model that was assumed in Section 2.

Now suppose that we fix on the positive-inflation expectations mapping, setting  $\phi = \Pi^{*-1}$ . As in Section 2 we can then investigate the conditional problem: *What is the set of temporary equilibria that could arise given this mapping?* It is trivial to see that there is just one possibility. From equation (16), we have that  $(1 + i_t) = \beta^{-1} \Pi^*$ , and this implies  $\Pi_t = \Pi^*$  from (17). Thus current inflation equals expected, consistent with steady state.

The main point is that in the canonical BSU setting, the problem *is* that there is a multiplicity in the set of expectations mappings. Fixing expectations to be consistent with the positive-inflation steady state is fully sufficient to select this as the unique possible outcome

in any given time period. The concern is that expectations may become trapped elsewhere, but so long as this is avoided, the ‘perils’ of Taylor rules are averted. Our paper focuses on an alternative result: there may be multiple equilibria associated with a given expectations mapping, including equilibria with large recessions, when the zero bound is present. For these to exist we need a richer economic environment than an efficient Walrasian model with no state variables, but this simple setting makes the conceptual distinction clear.

### 3.1 Learning, expectations mappings and sunspots

The distinction between multiplicity in the expectations mapping and multiplicity for a given expectations mapping matters because the most influential approach to select among equilibria in the BSU environment has been to invoke learning dynamics. Non-fundamental liquidity traps have the property that they are not locally E-stable, which means that simple adaptive learning rules will not induce convergence to them except when knife-edge initial conditions are assumed.<sup>20</sup> A number of authors have used this result to suggest that an expectations trap with  $\phi = \beta^{-1}$  should not be viewed as a likely outcome. Christiano and Eichenbaum (2012), for instance, dismiss it as a ‘mathematical curiosity’, whilst McCallum (2007) argues more generally that E-stability should be regarded as a ‘necessary condition for the relevance of a RE equilibrium’.

Clearly this is not universally accepted. Following Evans, Guse and Honkapohja (2008), Mertens and Ravn (2014) study a New Keynesian setting with adaptive learning and make the point that the time that it would take for beliefs to transition away from the neighbourhood of the low-inflation steady state could be very long.

Yet the more important point from the perspective of our paper is that learning processes are precisely methods for selecting a unique rational expectations mapping, not for choosing among multiple solutions once this mapping is fixed. In the general case they operate by specifying a conjectured learning rule linking expected future variables to current outcomes, of the form:

$$\mathbb{E}_t x_{t+1} = \phi_t(x_t^s) \tag{18}$$

where  $x_t$  is a vector of variables of interest in  $t$ , and  $x_t^s$  is a subset of these that is relevant to outcomes at  $t + 1$  – the model’s state variables.<sup>21</sup> The focus is on convergence in the mapping  $\phi_t$ , given some updating rule  $\phi_{t+1} = \mathcal{T}(\phi_t)$ . But the entire temporary equilibrium system comprises a vector of restrictions of the form:

$$F(x_{t-1}^s, x_t, \mathbb{E}_t x_{t+1}, \varepsilon_t) = 0 \tag{19}$$

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<sup>20</sup>This point is made, for instance, in Evans and Honkapohja (2005) and Evans, Guse and Honkapohja (2008).

<sup>21</sup> $x_t^s$  may also include the persistent component of sunspot processes.

where  $\varepsilon_t$  is a stochastic innovation in period  $t$ .

Convergence to a unique  $\phi$  implies that this system can be replaced by:

$$F(x_{t-1}^s, x_t, \phi(x_t^s), \varepsilon_t) = 0 \quad (20)$$

This is a system of equations in the vector of variables  $x_t$ , given  $x_{t-1}^s$  and  $\varepsilon_t$ . It is precisely the problem of finding a temporary equilibrium. In linear models there will generically be a unique solution, provided the number of equations stacked in  $F$  is equal to the number of variables in  $x_t$ .<sup>22</sup> But systems with the ZLB by definition feature a significant non-linearity, and for regions of the parameter space with positive Lebesgue measure there may be more than one solution to (20).<sup>23</sup> In particular, as highlighted in Section 2, sufficient endogenous persistence can ensure that a fall in current output raises the marginal value of saving, and this may mean that low current output is an equilibrium-consistent outcome.

When there is more than one solution to (20) there must be some coordinating device for selecting an equilibrium. In the analysis that follows we assume that this operates via a random public signal that is realised within time period  $t$ .

This random public coordination device shares some similarities with the modelling approaches taken by Mertens and Ravn (2014) and Aruoba and Schorfheide (2013). Both of these papers assume that an exogenous sunspot process coordinates outcomes on a particular dynamic equilibrium path, in models where a fundamental multiplicity of the sort analysed by BSU is present. The difference with our work is that these authors use a persistent two-state Markov sunspot process, whose current realisation is itself a state variable in the model. If we denote the sunspot by  $s_t$ , this means that the expectations mapping takes the form  $\phi(s_t)$ . Different realisations of  $s_t$  correspond to the economy being in a state of ‘optimism’ or ‘pessimism’ at a given point in time. A state of pessimism is associated with expectations of future deflation, and this implies current deflation and a binding zero bound. The approach effectively allows for the economy to switch between the two steady-state regimes identified by BSU, coordinated by the sunspot process.

Persistence in the sunspot shock is vital to this mechanism. Pessimism implies deflation because it is associated with expectations of deflation, and it is associated with expectations of deflation because the same pessimistic state – and deflationary outcome – is expected in the future with a sufficiently high probability.<sup>24</sup> If the sunspot variable were iid, the possibility of multiplicity would disappear. In such a case the expectations mapping would be independent of  $s_t$ . In the Walrasian example that we sketch above, any time-invariant

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<sup>22</sup>Solving for  $x_t$  in this case amounts to a matrix inversion problem. McCallum (2007) discusses the possibility of singularities, in which case there is indeterminacy. Yet this is a knife-edge outcome.

<sup>23</sup>That is, the result does not require a knife-edge choice of parameters.

<sup>24</sup>For instance in Mertens and Ravn (2014), pessimism persists with probability 0.7 from one quarter to the next.



expectations mapping would then be a constant, and we are back with the two possibilities,  $\phi = \Pi^{*-1}$  and  $\phi = \beta^{-1}$ .

The multiplicity that we focus on in this paper also relies on persistence, but of an endogenous form. As explained in Section 2, our focus is on models in which low current output delivers low expectations of future output and inflation, and we show that this endogenous form of persistence can be enough to generate self-fulfilling recessions. This means we do not need an exogenous sunspot process to link pessimism from one time period to the next. Where we make use of a randomisation device, it is assumed to be iid.

## 4 A non-linear model with randomisation

### 4.1 Overview

In this section we provide a computed numerical example of our self-fulfilling recessions, embedded in a recursive rational expectations equilibrium. We focus on a non-linear version of the canonical New Keynesian model, with just one departure from standard practice. This is that we assume the central bank has a Taylor-style policy rule that feeds back on the *growth rate* of output, rather than its level. When the degree of feedback on growth is high enough, this can be enough by itself to generate the sorts of self-fulfilling recessions that we identified in the partial equilibrium setting of Section 2. The intuition is that a collapse in output in the current period implies a reversion back to steady state in the future. This means that output is expected to grow in the future. The central bank's feedback policy will therefore mandate relatively tight future monetary policy, and this constrains future output and inflation rates. This is exactly the link from low current output to low future output and inflation that we need.

We choose to focus on this mechanism for three reasons. The first is for ease of exposition: it is the simplest and most transparent way that we have found to illustrate our main argument in a rational expectations, general equilibrium setting. By design our mechanism requires a model with nominal rigidities, so that the zero bound matters, and where low output can occur in equilibrium because of deficient aggregate demand. The New Keynesian framework allows both of these. But the standard New Keynesian model does not have a strong endogenous propagation mechanism. Price dispersion can be a state variable when Calvo price-setting is used, but this does not have a strong enough impact on equilibrium allocations for the sort of propagation that we need. There is no other state variable in the textbook version of the model. Introducing feedback on the output gap provides the persistence we need without expanding the model any further, and this allows our arguments to be understood in the textbook setting.

The second reason for focusing on this mechanism is that growth feedback has been found to play a significant role in the policy rule in estimated DSGE models. The posterior mode for the growth feedback coefficient estimated for the US economy in Smets and Wouters (2007), for instance, is 1.16, compared to a feedback coefficient on the level of GDP of just 0.08. Many subsequent papers have neglected to model feedback on the level of output, since the explanatory power of this channel seems weak. Yet, as we show in Section 5, having a policy rule of this form strongly affects the likelihood of self-fulfilling recessions being possible these models. We show that the Smets and Wouters (2007) and Iacoviello and Neri (2010) models place a high posterior probability on the region of the parameter space that is associated with multiplicity. But in both cases, if the policy rule is instead changed to one where feedback is on the level of the output gap, the probability of multiplicity is significantly reduced. This implies a clear policy lesson: output growth feedback, of an empirically plausible kind, makes economies vulnerable to self-fulfilling recessions. This section provides a clear account for why this is so, in a model with comparatively few dynamic frictions to complicate the interpretation.

This relates to the third reason for focusing on Taylor-style rules with growth feedback, which is that a number of authors have recommended such rules as normatively desirable. Two distinct arguments have been made. The first, due in particular to Athanasios Orphanides,<sup>25</sup> is based on the difficulty of measuring potential output. If one accepts that policy should follow *some* simple rule of the Taylor form, a rule that feeds back on the growth rate of output (or even the growth rate of the output gap) is less sensitive to mismeasurement than one that attempts to feed back on the level.

The second normative argument is that in a linearised New Keynesian model it can be optimal, or near-optimal, for the central bank to target a measure of the growth rate of the output gap over time. This is shown, for instance, in Woodford (2003) and Blake (2012).<sup>26</sup> The intuition relates to the well-known ‘stabilisation bias’ time inconsistency problem in the New Keynesian model.<sup>27</sup> Because current inflation depends negatively on expectations of the future output gap, it is desirable to respond to any current inflationary shock with a relatively tight monetary policy over number of time periods, rather than concentrating all stabilisation in the initial period of the shock. This means avoiding too rapid a return of the output gap to zero – something that can be implemented in practice by moderating the rate of change in output at any given point in time.

Overall, therefore, output growth feedback is something that seems empirically plausible

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<sup>25</sup>See, for instance, Orphanides (2001, 2003) and Orphanides and Williams (2002).

<sup>26</sup>Specifically, this a targeting procedure is the recommendation of a Ramsey-optimal plan for all time periods after the first, in a linear New Keynesian setting when policy seeks to maximise a quadratic approximation to a representative agent’s welfare.

<sup>27</sup>See Svensson (1997) and Clarida et al. (1999) for discussions of stabilisation bias.

as a description of how central banks *do* act, and is certainly defensible as a prescription for how they *should*. For this reason it is of interest to understand how far this sort of strategy implies a worsened exposure to the threat of self-fulfilling recessions once the zero bound is incorporated.

## 4.2 Model setup

We use a standard New Keynesian model without capital and with Calvo (1983) pricing. The basic elements are presented only. A representative consumer in period  $t$  chooses a consumption plan in order to maximise the discounted present value of their utility, given by:

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\sigma} - 1}{1-\sigma} - \frac{L_s^{1+\nu}}{1+\nu} \right] \quad (21)$$

where  $L_s$  gives labour hours and  $C_s$  is a CES aggregate over differentiated goods  $C_s(i)$ , living on the unit interval:

$$C_s := \left[ \int_0^1 C_s(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (22)$$

with  $\varepsilon > 1$  the elasticity of substitution across goods. Government expenditure is aggregated in an identical manner. Consumer choice is subject to the intertemporal budget constraint:

$$P_t C_t + B_{t+1} \leq W_t l_t + (1 + i_{t-1}) B_t + T_t \quad (23)$$

where  $P_t$  is the usual CES price index,  $B_{t+1}$  is the consumer's holdings of nominal bonds, which will pay  $(1 + i_t)$  monetary units with certainty at  $t + 1$ ,  $W_t$  is the nominal wage rate and  $T_t$  collects any period- $t$  profit income and lump-sum taxes/transfers from the government. The solution to this implies the usual Euler condition:

$$\beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + i_t)}{\Pi_{t+1}} \right] = 1 \quad (24)$$

where  $\Pi_{t+1}$  is the gross inflation rate in period  $t+1$ ,  $\frac{P_{t+1}}{P_t}$ .

The monopolistic producer of good type  $i$  employs a simple technology that is linear in labour:

$$Y_t(i) = L_t(i) \quad (25)$$

where  $Y_t(i)$  is output of firm  $i$  and  $L_t(i)$  its employment. The firm can reset its price each period with a constant probability  $(1 - \theta)$ . When possible, it chooses a price  $P_t^*(i)$  to maximise the discounted sum of current and future profits,  $V_t(i)$ . Firms are wholly owned by

consumers, so this is evaluated using the consumer's stochastic pricing kernel:

$$V_t(i) := \mathbb{E}_t \sum_{s=t}^{\infty} (\beta\theta)^{s-t} \left( \frac{C_s}{C_t} \right)^{-\sigma} \frac{1}{P_s} [P_t^*(i) - (1-\tau)W_s] Y_s(i) \quad (26)$$

where  $\tau$  is a per-hour hiring subsidy that is used to correct monopoly distortions.<sup>28</sup> Replacing  $Y_s(i)$  by the consumer's relative demand function and taking a first-order condition for optimal  $P_t^*$  gives an averaged markup pricing rule:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\theta)^{s-t} \left( \frac{C_s}{C_t} \right)^{-\sigma} Y_s(i) \left[ \frac{P_t^*(i)}{P_s} - \frac{\varepsilon}{\varepsilon-1} (1-\tau) \frac{W_s}{P_s} \right] = 0 \quad (27)$$

Using this it can be shown that aggregate inflation in period  $t$  satisfies the condition:

$$(1-\theta) \left( \frac{S_t}{F_t} \right)^{1-\varepsilon} + \theta \Pi_t^{\varepsilon-1} = 1 \quad (28)$$

where  $S_t$  and  $F_t$  are defined recursively by:

$$S_t = \frac{\varepsilon}{\varepsilon-1} (1-\tau) L_t^\nu Y_t + \theta \beta \mathbb{E}_t S_{t+1} \Pi_{t+1}^\varepsilon \quad (29)$$

$$F_t = C_t^{-\sigma} Y_t + \theta \beta \mathbb{E}_t F_{t+1} \Pi_{t+1}^{\varepsilon-1} \quad (30)$$

with  $Y_t$  the aggregate production level of the CES composite. Aggregate market clearing will require:

$$C_t + G_t = Y_t \quad (31)$$

where  $G_t$  is government expenditure.

Given the linear production function,  $Y_t$  will be given by the aggregate labour input  $l_t$ , corrected for inefficiencies due to price dispersion:

$$Y_t = \frac{L_t}{\Delta_t} \quad (32)$$

where the price dispersion term  $\Delta_t \geq 1$  can be shown to satisfy the recursion:

$$\Delta_t = (1-\theta) \left( \frac{S_t}{F_t} \right)^{-\varepsilon} + \theta \Pi_t^\varepsilon \Delta_{t-1} \quad (33)$$

Equations (24) and (28) to (33) give seven dynamic equilibrium conditions in the nine processes  $\{C_t, L_t, Y_t, i_t \Pi_t, \Delta_t, S_t, F_t, G_t\}$ . A policy rule for the nominal interest rate and some

<sup>28</sup>We follow the bulk of the literature in using this device, but our results are qualitatively and quantitatively very similar if the monopoly distortion remains uncorrected.

specification of government expenditure will be sufficient to close the model. As above, we will assume interest rates follow a feedback rule that responds to the growth rate of output, as this is the simplest way to generate multiplicity:

$$(1 + i_t) = \max \left\{ \beta^{-1} \Pi^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\alpha_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\alpha_y}, 1 \right\} \quad (34)$$

where  $\Pi^*$  is the target rate of inflation. Initially we assume that government expenditure is fixed at some value  $\bar{G}$ .

### 4.3 Equilibrium definition

We will focus our attention on recursive rational expectations equilibria in this setting. Given the earlier distinction between multiplicity in the expectations mapping and multiplicity for a given mapping, we take some time to formalise these concepts in the current context. Equilibrium will be defined by reference to a policy function for the model's endogenous variables, and an expectations mapping. The policy function takes the form:

$$x_t = g(Y_{t-1}, \Delta_{t-1}, \sigma_t) \quad (35)$$

where  $x_t$  denotes the vector  $[C_t, L_t, Y_t, i_t \Pi_t, \Delta_t, S_t, F_t, G_t]'$ , and  $\sigma_t$  is a binary random variable that takes value 1 with probability  $p(Y_t, \Delta_t)$ , and 0 with probability  $(1 - p(Y_t, \Delta_t))$ .<sup>29</sup> The expectations mapping is a relationship of the form:

$$\mathbb{E}_t f(x_{t+1}) = \phi(Y_t, \Delta_t) \quad (36)$$

where  $f(x_{t+1})$  stacks as a vector the set of expectations over  $t + 1$  outcomes relevant to equilibrium outcomes in  $t$ . In this case,  $f(x_{t+1}) = [C_{t+1}^{-\sigma} \Pi^{-1}, S_{t+1} \Pi_{t+1}^\varepsilon, F_{t+1} \Pi_{t+1}^{\varepsilon-1}]'$ , since these are the expectational variables that feature in the eight model equations: (24) and (28) to (34). Notice that expectations here depend on the endogenous state vector, but not directly on  $\sigma_t$ . This captures the idea that if there *is* more than one possible outcome in period  $t$ , this is not because of a direct shock to expectations themselves. This contrasts with the approach taken in Mertens and Ravn (2014) and Aruoba and Schorfheide (2013), where self-fulfilling recession dynamics require persistent sunspot processes that have a direct causal impact on expectations.

Equilibrium requires that conditions (24) and (28) to (34) should hold for all admissible realisations of the state vector. For notational convenience let these conditions be stacked in

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<sup>29</sup>Occasionally we will use subscripts to identify elements of  $g$  – for instance  $g_Y(Y_{t-1}, \Delta_{t-1}, \sigma_t)$  denotes the policy function for  $Y$  alone.

Parameter	Description	Value
$\beta$	Discount factor	0.995
$\sigma$	Inverse EIS	1
$\nu$	Inverse Frisch	2
$\varepsilon$	Elasticity of substitution	10
$\theta$	Calvo survival prob.	0.65
$\alpha_\pi$	Inflation feedback	1.5
$\alpha_y$	Growth feedback	3
$\bar{\Pi}^*$	Inflation target	1.005
$\tau$	Production subsidy	0.1
$\bar{G}$	Government spending	0.2

Table 2: Parameter values

the single vector restriction:

$$h(x_t, \mathbb{E}_t f(x_{t+1}), Y_{t-1}, \Delta_{t-1}) = 0 \quad (37)$$

As is standard, we consider values of the state variables  $(Y, \Delta)$  that live within some compact set, denoted  $\mathcal{S} \subset \mathbb{R}^2$ . The equilibrium concept is defined formally as follows:

**Definition.** The policy function  $g$  and expectations mapping  $\phi$  are together consistent with a recursive rational expectations equilibrium if and only if for all  $(Y_{t-1}, \Delta_{t-1}) \in \mathcal{S}$  and  $\sigma_t \in \{0, 1\}$ , the following three conditions hold:

1. [Equilibrium consistency]  $h(g(Y_{t-1}, \Delta_{t-1}, \sigma_t), \phi(g_Y(Y_{t-1}, \Delta_{t-1}, \sigma_t), g_\Delta(Y_{t-1}, \Delta_{t-1}, \sigma_t)), Y_{t-1}, \Delta_{t-1}, \sigma_t) = 0$
2. [Rational expectations]  $\phi(Y_t, \Delta_t) = p(Y_t, \Delta_t) f(g(Y_t, \Delta_t, 1)) + (1 - p(Y_t, \Delta_t)) g(Y_t, \Delta_t, 0)$
3. [Boundedness]  $\{g_Y(Y_{t-1}, \Delta_{t-1}, \sigma_t), g_\Delta(Y_{t-1}, \Delta_{t-1}, \sigma_t)\} \in \mathcal{S}$

The purpose of the variable  $\sigma_t$  is to coordinate economic outcomes on one of two equilibria: a ‘normal’ outcome in which output and inflation are moderate and the nominal interest rate is strictly positive, and a recessionary outcome in which the zero bound binds. In principle it may be the case that only one of these equilibria is possible for some values of the state vector. This can be allowed for by letting  $p(Y_{t-1}, \Delta_{t-1})$  take a value of either zero or one accordingly, for the relevant  $(Y_{t-1}, \Delta_{t-1})$  values.

## 4.4 Calibration

Our simulations assume the following parameter values:

Most of these values are standard in the literature. The model frequency is assumed to be quarterly, so the discount factor implies roughly a 2 per cent steady-state real interest rate, and the inflation target is 2 per cent per annum. Setting  $\bar{G}$  to 0.2 implies that the government would consume 20 per cent of GDP in the model's non-stochastic steady state, which corresponds approximately to federal expenditure levels in the United States.

The one non-standard choice is the value for  $\alpha_y$ , which is set to 3. This is higher than reported in most empirical studies, whether of the single-equation kind or based on a larger estimated DSGE model,<sup>30</sup> but such a choice is necessary in order for the model to exhibit the self-fulfilling recessions that are our focus. As explained in Section 2, in order for self-fulfilling recessions to occur we need a model with either a high degree of endogenous persistence in output, or a strong link from low current output to low future inflation, or some combination of the two. It is well known that the New Keynesian model has a very weak endogenous propagation mechanism. Absent feedback on growth, the only state variable is the price dispersion term  $\Delta_t$ , and this does not have a significant impact on equilibrium outcomes so long as inflation remains modest. This means that almost all of the propagation in the model that we simulate will be injected via the policy rule. Accordingly, in order to ensure that persistence-induced multiplicity is present in this simple setting, we need the policy response to growth to be sufficiently strong.

Thus if  $\alpha_y$  appears excessive in magnitude, it should be remembered that it needs to be so large only because the rest of the model is not providing any significant additional source of persistence. In Section 5 we turn our attention to models whose persistence properties are a better fit to business cycle data, and find that such high growth feedback is no longer necessary. A practical difficulty with these models is their large number of endogenous states. It is not feasible to solve them using the global methods that we deploy in this section, so the analysis relies instead on quasi-linear techniques of the sort introduced by Eggertsson and Woodford (2003). The analysis in the present section shows that the basic mechanism operates in a fully non-linear setting.<sup>31</sup>

Numerically we found that the key threshold for multiplicity to exist was  $\alpha_y > \sigma\alpha_\pi$ ,<sup>32</sup> but for identical reasons to those illustrated in Figure 2, the closer parameters are to this threshold, the more severe are the associated recessions. For this reason we chose the value

<sup>30</sup>Orphanides (2003b) reports values of  $\alpha_y$  in the region of 0.5 based on single-equation estimates from post-1980s US data. Smets and Wouters (2007) find  $\alpha_y = 1.16$  at the posterior mode, significantly higher than their prior mean.

<sup>31</sup>There has been some disagreement in the literature on the appropriateness of adding a zero bound into models that are otherwise linearised. Braun et al. (2012) and Fernandez-Villaverde et al. (2015) indicate the possible inaccuracies that may result. Christiano and Eichenbaum (2012) argue nonetheless that headline results on the government spending multiplier obtained in linear models are not affected by adopting a global solution technique.

<sup>32</sup>An earlier working paper provided a formal proof that this condition was necessary and sufficient for multiplicity in a linearised version of the model.

of 3, which is comfortably within the necessary range.

#### 4.4.1 The probability of recession

There is no equilibrium restriction pinning down the probability function  $p(Y, \Delta)$ . The only restriction that is placed on the function is a need for equilibrium consistency, meaning that it can only attach positive probability both to zero bound episodes and to ‘normal’ equilibria if multiplicity is indeed a possibility, which in turn depends on the equilibrium expectations mapping. Since the expectations mapping is constructed for a given  $p$ , this consistency requirement has a fixed-point character, and needs to be verified *ex-post*. In practice we found that the chosen calibration allowed multiplicity to be generated with an invariant function  $p(Y, \Delta) = \bar{p}$  for all  $(Y, \Delta)$  values in the grid space. For our benchmark exercise we set the probability  $\bar{p}$  to 0.02, consistent with a self-fulfilling recession being generated roughly every 50 quarters.

A range of alternative values is possible, and variations in the  $p$  function that is specified will change the model’s predictions via an expectations channel – both during a recession and in normal times. This means that there is an indeterminacy in the model, which follows as a direct consequence of the finite multiplicity problem. This feature has a parallel in the recent literature that builds on Benhabib et al (2001), where the Markov transition matrix for the sunspot process that governs expectations must be exogenously fixed.

### 4.5 Solution method

We solve the model by iterating on an expectations mapping  $\phi$ , defined on a grid of values for  $Y$  and  $\Delta$ . Starting with a given  $\phi$ , we solve at every point on the grid for two equilibrium outcomes: one in which the zero bound binds, and one in which it does not. In each case we verify that the resulting outcome is consistent with the policy rule (34). That is, when the zero bound is imposed, we check that the implied inflation and output growth rates are indeed consistent with a binding ZLB, and when it is not imposed we check that the chosen nominal interest rate is positive. Given these outcomes and an exogenously specified  $p(Y, \Delta)$  function, the expectations mapping can then be updated. The process is repeated to convergence.

### 4.6 Results

#### 4.6.1 Benchmark exercise: income versus substitution

Figure 3 charts the dynamics of output, inflation, the expected real interest rate and the nominal interest rate during a self-fulfilling recessionary episode under the assumed parameter



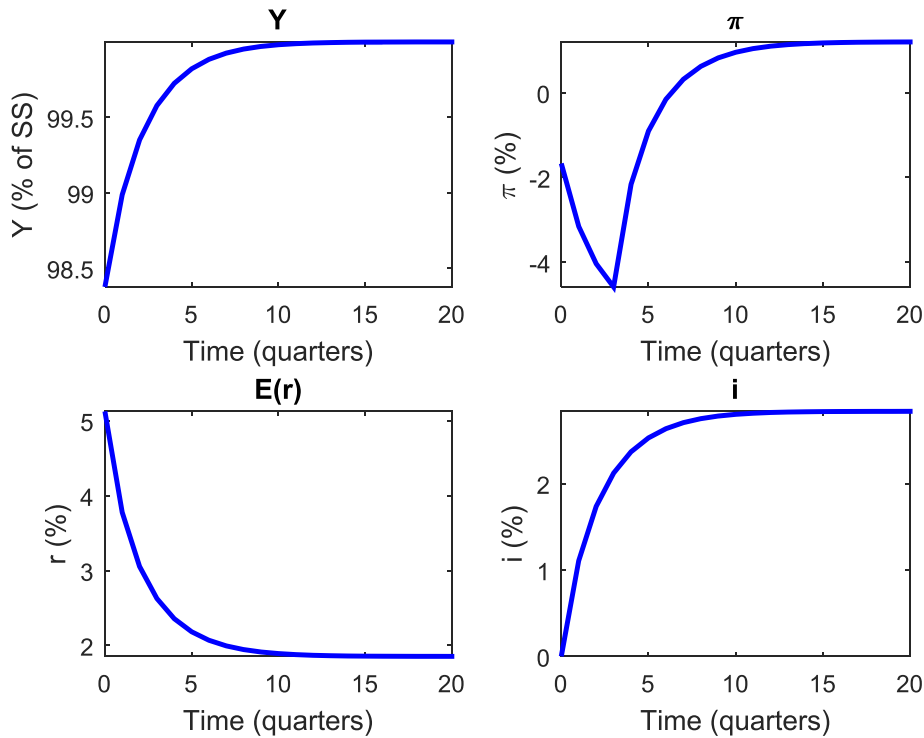


Figure 3: Impulse responses: a self-fulfilling recession

values.<sup>33</sup> The charts are constructed by assuming a realised series for the coordinating variable  $\sigma_t$  of an initial 1 followed by 0 in all periods thereafter. There is no other shock driving this outcome aside from  $\sigma_t$ .

The general pattern is of a textbook demand-driven recession: there is a short, sharp decline in output and prices, with nominal interest rates reaching the zero bound but real rates nonetheless spiking – driven by decreased inflation expectations. The figure is drawn assuming an initial state vector  $(Y_{t-1}, \Delta_{t-1})$  that is consistent with a long previous history without any recessionary episodes, but the dynamics are not significantly affected by starting elsewhere.<sup>34</sup> Irrespective of the state vector, the economy’s stay at the zero bound lasts just one period (unless a further recessionary episode hits immediately – i.e.,  $\sigma = 1$  in two consecutive periods). This is a reflection of the relatively weak propagation mechanism that is present in this model, deriving almost entirely from the lagged output term in the policy rule. In richer economies it seems possible that the zero bound may bind for many periods, as the cases studied in Section 5 show.

<sup>33</sup>Inflation is given as the year-on-year percentage change in the price index, assuming normal times prior to the episode. Interest rates are annualised. Output is measured in percentage deviations from its long-run level when no sunspot shocks arise.

<sup>34</sup>Note that conditional on the zero bound binding, the lagged value of output is irrelevant to the equilibrium allocation, since it only enters via the (redundant) policy rule. This means it is only the initial value of  $\Delta_{t-1}$  that can affect recessionary dynamics, and its impact in practice is weak.

The partial equilibrium example in Section 2 focused on the role of two distinct expectations effects in ensuring self-fulfilling recessions were possible. The first was an income effect: given persistence in economic conditions, a current recession implies low future earnings, and thus low current demand. The second was a substitution effect: low current output implies low future inflation, and thus a high real interest rate – again reducing current demand. In that example the strength of the two channels was fixed exogenously by the size of the autoregressive coefficient  $\rho$  (in the case of the income effect), and the size of the output-inflation propagation parameter multiplied by the EIS,  $\frac{\delta}{\sigma}$  (for the substitution effect). The key threshold for multiplicity to be possible was  $\rho + \frac{\delta}{\sigma} > 1$ .

In the present example there will not be such a straightforward parametric decomposition, but it is possible to provide an exploratory quantification of the two effects that operates along similar lines. We do this by reference once again to the Euler condition:

$$C_t^{-\sigma} = \beta (1 + i_t) \mathbb{E}_t \left[ C_{t+1}^{-\sigma} \Pi_{t+1}^{-1} \right] = 1 \quad (38)$$

As explained in Section 2, self-fulfilling recessions will occur when the elasticity with respect to  $Y_t$  of the expectations term on the right-hand side of (38) exceeds the elasticity with respect to  $Y_t$  of the term in  $C_t$  on the left-hand side. When this is true, decreases in current output increase the relative benefits from current saving, notwithstanding the fact that current consumption is also more valuable. Since we have multiplicity, we know that overall this requirement is met here. Now we ask the further question: Suppose that counterfactually we were to switch off the income (substitution) channel. How far can the substitution (income) channel in isolation take us towards multiplicity?

To answer this we seek numerical equivalents to the values of  $\rho$  and  $\delta$  in the current context. We conduct the following experiment. Suppose that, counterfactually,  $\Pi_{t+1}$  were known with certainty at  $t$ , irrespective of  $Y_t$ . This means that the substitution effect has been switched off. The Euler equation could then be rewritten:

$$C_t = \left[ \beta (1 + i_t) \Pi_{t+1}^{-1} \right]^{-\frac{1}{\sigma}} \left[ \mathbb{E}_t \left[ C_{t+1}^{-\sigma} \right] \right]^{-\frac{1}{\sigma}} \quad (39)$$

The term  $\left[ \mathbb{E}_t \left[ C_{t+1}^{-\sigma} \right] \right]^{-\frac{1}{\sigma}}$  is a generalised mean value for  $C_{t+1}$ . From the model's policy function for consumption we can investigate in isolation how this object changes as  $Y_t$  changes.<sup>35</sup> Since government spending is held constant, this can equivalently be viewed as a measure of the responsiveness of the generalised mean value of  $C_{t+1}$  to changes in  $C_t$ . When the variables are compared in log form this responsiveness corresponds precisely to the earlier

<sup>35</sup>This is necessarily an illustrative off-equilibrium exercise: not every value of  $Y_t$  is possible in equilibrium. In carrying out the exercise we need to make some assumption about the response of  $\Delta_t$  to any change in  $Y_t$ , and for this we simply assume that  $\Delta_t$  is held constant at its long-run steady-state level.

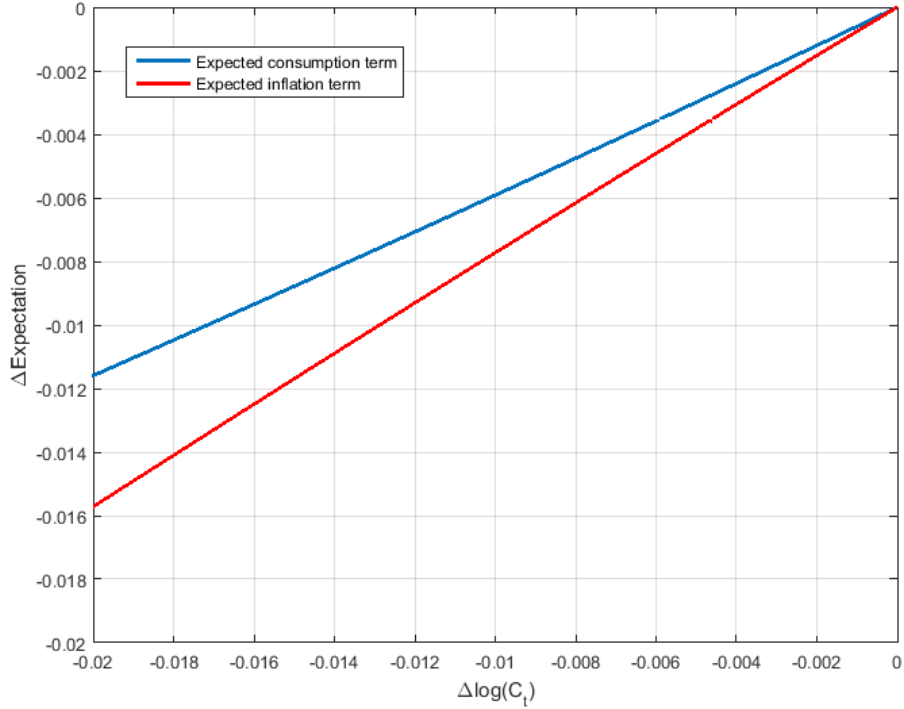


Figure 4: Measuring the income and substitution channels

autoregressive coefficient  $\rho$  – though in this case there need not be a constant  $\rho$ .

A similar procedure can be applied to measure the substitution channel. This means charting the elasticity of  $\left[\mathbb{E}_t \left[\Pi_{t+1}^{-1}\right]\right]^{-\frac{1}{\sigma}}$  in  $C_t$ , which by a similar argument is the equivalent here of  $\frac{\delta}{\sigma}$ .

Notice that this decomposition between channels is not additive. The covariance between future consumption and future inflation provides a further incentive to save that we will not capture. Nonetheless, it is insightful as an approximate guide. Figure 4 plots the quantities of interest: the departure from steady state in the log of  $\left[\mathbb{E}_t \left[C_{t+1}^{-\sigma}\right]\right]^{-\frac{1}{\sigma}}$  (blue) and the log of  $\left[\mathbb{E}_t \left[\Pi_{t+1}^{-1}\right]\right]^{-\frac{1}{\sigma}}$  (red) as the log of  $C_t$  is reduced below its ‘normal equilibrium’ steady-state value.<sup>36</sup> We are interested in the relative magnitudes of the slopes of these lines, since these are the equivalents of the elasticities  $\rho$  and  $\frac{\delta}{\sigma}$  respectively.

As the figure shows, it is the substitution effect that appears to play a greater role in inducing higher savings than the income effect. Graphically this follows from the fact that the red line is uniformly below the blue. For every percentage-point fall in  $C_t$  there is around a 0.8 percentage-point fall in  $\left[\mathbb{E}_t \left[\Pi_{t+1}^{-1}\right]\right]^{-\frac{1}{\sigma}}$ , whilst the fall in  $\left[\mathbb{E}_t \left[C_{t+1}^{-\sigma}\right]\right]^{-\frac{1}{\sigma}}$  is in the region

<sup>36</sup>That is, below the value for  $\log C_t$  that obtains in the equilibrium with no binding zero bound, when no recession has hit for a large number of periods.

of 0.55 to 0.6 percentage points. Loosely, therefore, the model is capturing an autoregressive coefficient in log consumption of 0.6 at most, but a much stronger pass-through from current output to future inflation. Thus when output falls it appears that individuals' marginal willingness to save increases principally through an intertemporal substitution channel. As reported in Section 2, business cycle statistics indicate a stronger role instead for the income channel. A challenge for future work will be to build this in to a tractable nonlinear model.

#### 4.6.2 Fiscal policy

An important focus of the literature on macroeconomic outcomes at the zero bound has been the potential for fiscal policy to mitigate recessions. The influential papers by Christiano, Eichenbaum and Rebelo (2011) and Eggertsson (2011) have shown that the government expenditure multiplier can be very large in a 'fundamental' liquidity trap – i.e., one that arises due to a negative shock to the natural rate. The main reason for this is that a promise to keep government spending high so long as the liquidity trap lasts – the experiment that the two papers consider – has a positive impact on inflation expectations, and thus stimulates current spending through an intertemporal substitution channel.<sup>37</sup> In their benchmark experiment in a model without capital, Christiano et al. find a government expenditure multiplier of 3.7.

Against this, Mertens and Ravn (2014) have shown that the multiplier is less than one, in a non-fundamental liquidity trap of the BSU type. The reason for this is that a confidence trap relies on a large negative output gap in order for deflation to persist. Were output to increase one-for-one with government spending, the result would be additional inflationary pressure. To offset this, and preserve the high real interest rate that the liquidity trap demands, output must fall back somewhat – ultimately increasing less than one-for-one with government spending.

In this subsection we conduct an identical experiment to Christiano et al. (2011) and Mertens and Ravn (2014), raising government expenditure above  $\bar{G}$  for so long as the zero bound remains binding. We show that *conditional on a self-fulfilling recession being realised*, its magnitude is now greater, and this can be interpreted as a negative fiscal multiplier. In this regard our results are a stronger counterpoint to the conventional finding of large multipliers even than Mertens and Ravn (2014). Yet we argue that this conclusion ought to be heavily qualified.

The precise exercise we conduct is to increase  $\bar{G}$  by 1 per cent of its value, from 0.2 to 0.202, whenever the zero bound on nominal interest rates is binding. Figure 5 compares the infla-

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<sup>37</sup>Rendahl (2016) shows that with persistent unemployment higher government spending may further boost current consumption in a liquidity trap via an income effect. By keeping workers employed, the government raises expected future earnings, and this increases current consumption.

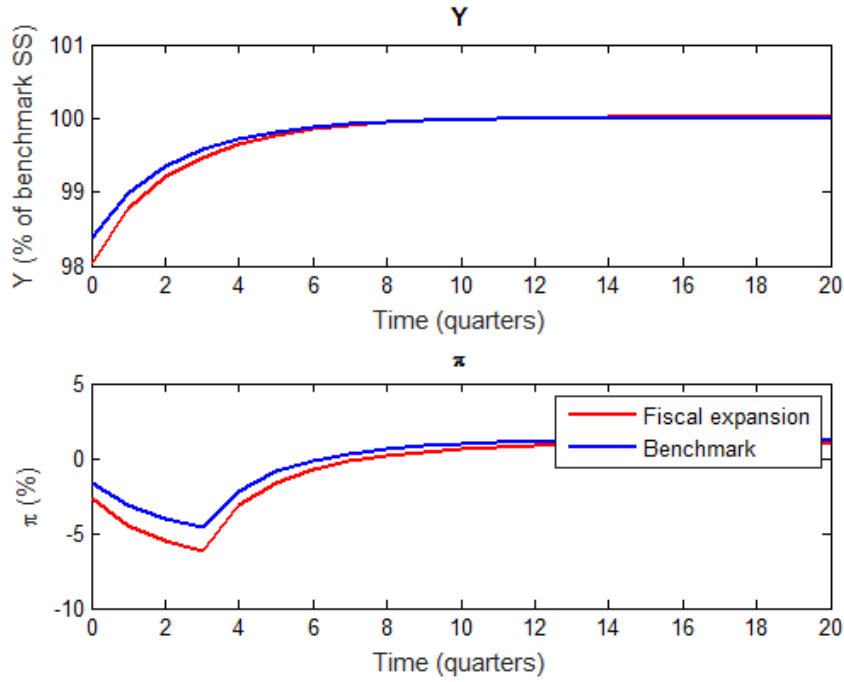


Figure 5: The contractionary effects of raising government spending

tion and output dynamics during a self-fulfilling recession when this assumption is made, against the benchmark of no fiscal expansion. The impact of the additional expenditure is to worsen the initial scale of the recession by almost half a percentage point of steady-state output, and to increase significantly the level of deflation that the economy experiences.

These effects can be translated into fiscal multipliers. Let  $\{Y_t\}_{t=0}^{\infty}$  be the output series that is generated in the benchmark policy model when a recession hits in period zero (and never thereafter), and let  $\{Y_t^f\}_{t=0}^{\infty}$  be the corresponding series when government spending increases in a downturn. We define the impact multiplier as  $\frac{Y_0^f - Y_0}{\Delta \bar{G}}$ , where  $\Delta \bar{G}$  is the increase in government spending during recessions. For a cumulative multiplier we focus on a 10-quarter horizon, by which time  $Y_t$  and  $Y_t^f$  have become nearly identical in value. Thus we define the cumulative multiplier as  $\frac{\sum_{t=0}^{10} (Y_t^f - Y_t)}{\Delta \bar{G}}$  – the total change in output relative to the total change in government spending across all time periods. Note that since the change in government spending is only realised so long as the zero bound binds, and since the zero bound only binds for one period, it is inevitable in this case that the cumulative multiplier will be greater in absolute magnitude than the impact multiplier.

Overall we find an impact multiplier of  $-1.86$  and a cumulative multiplier of  $-5.31$ . Clearly these are very large negative values, well outside of the range usually considered plausible in the literature. The intuition behind the results follows again from the Euler equation, and the relationship between the marginal incentive to consume and save, as dis-

cussed in Section 2. Consider a fall in current output of the of the same magnitude that is observed during a self-fulfilling recession in the benchmark model. For simplicity, denote the implied output level  $\hat{Y}_t$ . In the benchmark case, given the aggregate resource constraint, the consumption level that corresponds to this is  $\hat{Y}_t - \bar{G}$ . With a fiscal expansion, it is instead  $\hat{Y}_t - \bar{G} - \Delta\bar{G}$  – a lower value. *Ceteris paribus*, this increases the marginal utility value of current consumption relative to the marginal utility of savings, given that output has fallen to  $\hat{Y}_t$ . But the recessionary equilibrium is reached at an output level where the marginal value of current consumption equals the marginal value of saving. The point is that a larger output drop is now required for this to be true.

Superficially these results could be interpreted as suggesting that fiscal policy is extremely detrimental in a self-fulfilling recession. This seems too strong a conclusion to draw, for at least two reasons. First, a change in the government’s fiscal policy stance could simultaneously change the probability of entering a recession,  $\bar{p}$ . This probability is entirely indeterminate in our model, but in practice a perception that the government stands ready to act in the face of a confidence crisis could itself reduce (or increase) the likelihood that a collapse in output takes place. Second, and more significantly, we found that when the promised increases in  $\bar{G}$  at the zero bound were large enough, the recessionary equilibria ceased to exist. This implies fiscal policy can play a very positive role in ruling out self-fulfilling crises, provided it is sufficiently aggressive.

## 5 How likely is the multiplicity region?

The example of Section 4 demonstrated that the problem we identified in Section 2 could indeed manifest itself in a fully-specified recursive rational expectations equilibrium, solved in a non-linear way. Yet it is still an open question whether self-fulfilling recessions are possible for empirically plausible regions of the parameter space. As we have argued, the mechanism relies on a sufficiently large endogenous propagation mechanism, ensuring that low current output raises the current incentives to save. In Section 4 we injected this persistence directly into a standard New Keynesian model via a large feedback coefficient on output growth in the policy rule. The textbook New Keynesian setting provided simplicity and transparency, but the lack of a strong endogenous propagation mechanism in the underlying model meant that we needed to assume an empirically implausible value for this growth feedback coefficient. In this section we investigate whether the likelihood of multiplicity increases in less stylised models that contain a richer, and arguably more realistic, endogenous propagation mechanism.

## 5.1 Methodology

Our approach is to take ‘off the shelf’ two linearised DSGE models that have been widely cited in the literature, Smets and Wouters (2007) and Iacoviello and Neri (2010). We replicate the Bayesian estimation of these models’ parameters, and in each case we then investigate the size of the posterior likelihood that is attached to the multiplicity region of the parameter space.

For space reasons we provide a only brief verbal description of the two models here. Smets and Wouters (2007) is extremely well known, and has provided the core components for a huge number of subsequent extensions. It is a representative agent economy with an augmented version of the New Keynesian Phillips Curve and sluggish wage adjustment, together with a number of adjustment frictions in factor markets, and habits in consumption. Altogether it features nine endogenous states, not including exogenous shock processes, which makes for a very rich endogenous propagation mechanism. Building on earlier work in the DSGE literature by Christiano, Eichenbaum and Evans (2005) and others, it delivers impulse responses to monetary policy shocks that mirror the prolonged, ‘hump-shaped’ dynamics in output and inflation that are familiar from the SVAR literature.

Iacoviello and Neri (2010) develop a larger model with two classes of household, patient and impatient – a device that is used to ensure net lending in equilibrium.<sup>38</sup> They introduce a construction and a housing sector to the economy, and allow a credit channel to influence the effect of macroeconomic shocks, via changes in the collateral value of real estate. The purpose of this is to understand in more detail the role of the housing sector in macroeconomic dynamics – an important consideration that was commonly overlooked prior to 2008. The model additionally contains all of the ‘standard’ frictions that are modelled by Smets and Wouters (2007). Again, the scope for endogenous propagation is substantial: there are fifteen endogenous states, and variables respond in a prolonged, non-monotone fashion to the various different impulses.

We obtain estimates for the posterior distribution of parameters in each of the two models using publicly available replication codes, without introducing a zero bound. Since the models are estimated on a pre-crisis sample of US macroeconomic data without any zero bound episodes, the omission is not significant. The general strategy is then to take draws from the posterior distribution, and at each draw to check whether the resulting model is consistent with multiplicity or not. We ignore shocks, which are not of first-order relevance to the multiplicity question, and will consider the limiting case where agents place zero probability on the likelihood of a self-fulfilling recession – equivalent to  $\bar{p} = 0$  in the model of Section 4. In practice this means that we will be asking whether there is more than one perfect-foresight equilibrium path for any given initial state vector, holding constant long-

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<sup>38</sup>The paper incorporates the mechanism of Iacoviello (2005) into an estimated DSGE model.

run expectations. It is a simplification that allows piecewise-linear solution techniques to be applied.

Neglecting for now the zero bound, the models are of the standard linear form:

$$A_0 X_t = A_1 X_{t-1} + A_2 \mathbb{E}_t X_{t+1} \quad (40)$$

where  $X_t$  is the vector of endogenous variables and the  $A$  matrices contain the relevant interactions of the model's deep parameters.

To proceed we first solve for the state-space representation of the linear model, which takes the form:

$$X_t = \Phi X_{t-1} \quad (41)$$

for some matrix  $\Phi$ . This representation is unique, provided the usual Blanchard-Kahn conditions are met. Absent any complications from the zero bound,  $\Phi$  corresponds directly to the model's expectations mapping. In particular, this means that if it is known the zero bound will not bind from period  $t + 1$  onwards, the term  $\mathbb{E}_t X_{t+1}$  in (40) can be replaced by  $\Phi X_t$  – irrespective of whether the zero bound is binding in  $t$ . Thus the model's solution at  $t$  can be written as:<sup>39</sup>

$$X_t = [A_0 - A_2 \Phi]^{-1} A_1 X_{t-1} \quad (42)$$

In the event that the zero bound binds, the model takes an augmented form, which we can write as follows:

$$\tilde{A}_0 X_t = \tilde{A}_1 X_{t-1} + A_2 \mathbb{E}_t X_{t+1} + A_3 \quad (43)$$

where  $\tilde{A}_0$  and  $\tilde{A}_1$  are identical to their counterparts in equation (40) except in entries corresponding to the policy rule, where the linearised equivalent of  $i_t = 0$  is used in place, and  $A_3$  is a vector of zeros except for the entry corresponding to this policy rule.<sup>40</sup>

To implement the zero bound we use a 'once-and-for-all' approach, of the sort pioneered by Eggertsson and Woodford (2003). Let there be a certain time period  $T$  when the zero bound ceases to bind, and that once it has ceased to bind it is expected never to do so again. This means that in period  $T - 1$ , when the zero bound still binds, the equilibrium system can be written as:<sup>41</sup>

$$X_{T-1} = [\tilde{A}_0 - A_2 \Phi]^{-1} [\tilde{A}_1 X_{T-2} + A_3] \quad (44)$$

The solution is thus of the form  $X_{T-1} = \tilde{\Phi}_{T-1} X_{T-2} + \tilde{\Phi}_{T-1}^c$ , where  $\tilde{\Phi}_{T-1}$  is a matrix and

<sup>39</sup>The matrix  $[A_0 - A_2 \Phi]$  must be invertible for this representation. Numerically we have never encountered any examples of singularity.

<sup>40</sup>Recall that the linearised model considers the deviation of the nominal interest rate from its steady-state value, which means that the zero bound is implemented by fixing the *deviation* of  $i_t$  equal to some negative constant.

<sup>41</sup>Again the matrix  $[\tilde{A}_0 - A_2 \Phi]$  must be invertible here, and this was never a constraint in implementation.



$\Phi_{T-1}^c$  is a vector of constants. By iterating on the same approach we can obtain solutions for  $X_{T-2}$  in terms of  $X_{T-3}$ , and so on.

The model's expectations mapping in period 0 will be piecewise linear. To see this, suppose that the model starts in period 0 with an initial set of state variables  $X_{-1}$  that are consistent with steady state. Since there are no shocks, it is equilibrium-consistent for variables to remain in steady state permanently. Since this means the zero bound will not bind in period 1, local changes in  $X_0$  will change expectations at 0 according to  $\mathbb{E}_0 X_1 = \Phi X_0$ . For larger changes in  $X_0$ , however, it can come to be the case that the solution for  $X_1$  given by (42) would violate the zero bound in period 1. If this is true, expectations in period 0 will be consistent with a solution for  $X_1$  of the form  $\mathbb{E}_0 X_1 = \tilde{\Phi}_1 X_0 + \tilde{\Phi}_1^c$ , where the  $\tilde{\Phi}$  matrices are solved recursively given some  $T \geq 2$ . In general larger deviations of  $X_0$  from steady state imply a longer subsequent bind – that is, a higher value for  $T$ .

An recessionary equilibrium in period 0 is a value for  $X_0$  such that the zero bound is rationally expected to bind from period 0 until some period  $T - 1$ , so that  $X_0$  solves:

$$X_0 = [\tilde{A}_0 - A_2 \tilde{\Phi}_1]^{-1} [\tilde{A}_1 X_{-1} + A_2 \tilde{\Phi}_1^c + A_3] \quad (45)$$

and, in addition, the value of  $X_0$  implied by this equation must be such that the zero bound is indeed binding in period 0.<sup>42</sup>

For each drawing from the parameter space, our approach is therefore to conjecture values of  $T$ , and to investigate at each value whether there is a perfect-foresight equilibrium path such that the zero bound binds from 0 until period  $T - 1$ , given an initial state vector  $X_{-1}$  that is consistent with steady state.<sup>43</sup> If there is a value for  $T$  such that this alternative trajectory is equilibrium-consistent, we record that multiplicity is possible at the chosen parameter draw: a self-fulfilling recession is possible.

## 5.2 Results

Our headline results are summarised in the first column of Table 3.<sup>44</sup> In the Smets and Wouters (2007) model we find multiplicity at nearly every parameter draw, and in the Iacoviello and Neri (2010) model we find it at over two-thirds of draws. In both cases, and in contrast with the model of Section 4, the recessionary equilibria involve long stays at the zero bound – up to 20 quarters, depending on the particular parameter draw. We interpret

<sup>42</sup>In practice it must likewise be confirmed that the zero bound imposed from period 1 to  $T - 1$  is also binding.

<sup>43</sup>Since all variables are log deviations from steady state, this means  $X_{-1} = 0$ .

<sup>44</sup>To maximise accuracy, particularly given the high multiplicity probability obtained for the Smets and Wouters model, we base these estimates on the entire set of parameter draws generated by the MCMC algorithm during estimation, aside from the burn-in observations that are dropped. This amounts to 400,000 observations in each case.

Model	Multiplicity (benchmark)	Multiplicity (augmented rule)
SW (2007)	0.998	0.613
IN (2010)	0.686	0.056

Table 3: Multiplicity probabilities

this as clear evidence that when a macroeconomic model contains enough channels to generate empirically realistic degrees of persistence and propagation, the model is very likely to be exposed to the sort of mutliplicity that we have highlighted.

In both models we found that an important contributory factor to the likelihood of multiplicity was the monetary policy rule. Ignoring shocks, Smets and Wouters (2007) assume a rule of the following form:

$$r_t = \rho r_{t-1} + (1 - \rho) \{ r_\pi \pi_t + r_y (y_t - y_t^p) + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] \} \quad (46)$$

where  $r_t$  here is the nominal interest rate in period  $t$ , and  $y_t^p$  is a time-varying measure of potential output. Thus there is feedback on the growth rate of (potential) output, controlled by the coefficient  $r_{\Delta y}$ . As discussed above, the paper's central estimates for  $r_{\Delta y}$  are around 1.16, compared to feedback on the level of output,  $r_y$ , of just 0.08.<sup>45</sup> In Iacoviello and Neri (2010), the policy rule is of the simpler form:<sup>46</sup>

$$r_t = \rho r_{t-1} + (1 - \rho) \{ r_\pi \pi_t + r_{\Delta y} (y_t - y_{t-1}) \} \quad (47)$$

Again there is feedback on output growth, with the parameter  $r_{\Delta y}$  estimated to be around 0.52 in this case.<sup>47</sup>

We investigate the implications for multiplicity of 'switching off' growth feedback in the policy rule. To do this we draw again from the estimated parameter distribution, but instead of checking the benchmark model for multiplicity we check an augmented model where the policy rules are, respectively:

$$r_t = \rho r_{t-1} + (1 - \rho) \{ r_\pi \pi_t + (r_y + r_{\Delta y}) (y_t - y_t^p) \} \quad (48)$$

<sup>45</sup>Note that Smets and Wouters (2007) report as  $r_{\Delta y}$  a parameter on a growth feedback term that enters the policy rule outside of the brackets in (46), and for this reason takes a lower numerical value.

<sup>46</sup>Again we neglect shock terms. Because of the multi-sector nature of their model, Iacoviello and Neri (2010) distinguish between GDP and final goods output in their model, with housing investment also contributing to the former. The feedback in the policy rule is on GDP growth.

<sup>47</sup>The lower estimate by comparison with Smets and Wouters (2007) may partly be explained by Iacoviello and Neri placing a mean-zero prior on  $r_y$ , whereas Smets and Wouters assume a positive mean value.

in the Smets and Wouters model, and:

$$r_t = \rho r_{t-1} + (1 - \rho) \{r_\pi \pi_t + r_{\Delta y} \Delta y_t\} \quad (49)$$

in the Iacoviello and Neri model. That is, in each case we assume that the feedback parameter on growth is changed to a feedback parameter just on the level of output. In the Smets and Wouters model, where levels feedback was already present, the two coefficients are summed. Thus the experiment answers the question: ‘Suppose that monetary policy were refocused to ensure that it responds only to levels deviations in output from steady state. What is the impact on the possibility of self-fulfilling recessions occurring?’

The second column of Table 3 gives the likelihood of the multiplicity region in the two models when this alternative assumption is made about policy. In both cases the reduction is substantial. For the Smets and Wouters model multiplicity remains more likely than not, but it is no longer a near-certainty. For the Iacoviello and Neri model the likelihood drops close to 5 per cent. Clearly the sort of dynamic that was highlighted in Section 4 does play a significant role in allowing multiplicity to arise in larger models.

### 5.3 Qualification: the scale of recessions

There is an important qualification to our results in this section, and this is that in both models the magnitude of the self-fulfilling recessions that we find is very large. For the Smets and Wouters (2007) model, for instance, we find falls in the major series that are of a Great Depression order of magnitude. Figure 6 illustrates this, charting the percentage deviations of output, consumption, investment and the real wage from their steady-state values during a self-fulfilling recession, assuming values for the parameter vector equal to the posterior mode.

These provide a challenge for the analysis not because one would wish a priori to rule out the possibility of self-fulfilling recessions being very large, but because movements in macroeconomic aggregates on this scale are not consistent with the linear approximation that was used to write the underlying model in the first place. Nonetheless, a number of recent papers have applied New Keynesian liquidity trap analysis precisely to analyse policy changes during the Great Depression, and rely on a similar linearisation technique.<sup>48</sup>

## 6 Conclusion

The general message from this paper is that macroeconomic models with sufficiently rich propagation mechanisms can ‘take themselves’ to the zero bound, without any exogenous

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<sup>48</sup>See, for instance, Eggertsson (2008), Eggertsson (2012) and Roulleau-Pasdeloup and Zhutova (2015).

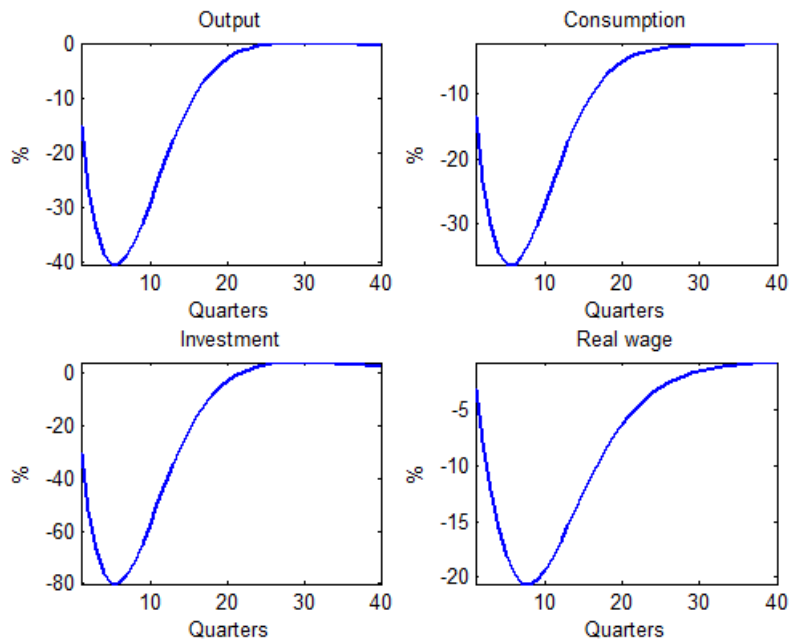


Figure 6: Large recessions in the Smets and Wouters (2007) model

impulse. This happens because a collapse in current output causes rational agents to expect low future output and low future inflation, which together raise the marginal incentives to save. Without a zero bound the monetary authority would be able to offset this effect by cutting rates, preventing any multiplicity problem. But when there is a zero bound, sufficiently large falls in current output can be supported in equilibrium, with the marginal incentive to save and the marginal incentive to consume just balanced.

An obvious question that this result throws up is whether existing analyses of recessions at the zero bound that rely on shock impulses could generate the same, or similar, results without any shocks. In many models a sufficiently large shock to the natural rate of interest is enough to ensure that there no longer exists an equilibrium in which the zero bound does not bind, and so an equilibrium path in which the bound is binding becomes the focus of attention. If the resulting dynamics are severe, this can be interpreted as the detrimental consequences of the discount factor shock. But our results suggest that in some circumstances exactly the same equilibrium exists regardless of whether or not the shock were to hit. If this were true it would be far more difficult to attribute the dynamics *causally* to a shock process that has really served only as an equilibrium selection device (by ruling out the main, tranquil alternative). These seem interesting avenues for further research.

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